The University of Calgary
Department of Mathematics and Statistics
MATH 249//01
Quiz # 4W

FALL 2007

Name:____________________________**I.D.#:**_________________

1. Find an equation of the tangent to

$$xy^2 - y = \frac{4x^3}{y} + 8$$

at the point $(1, -2)$. [5]

2. For $f(x) = \dfrac{4\sqrt{x} + x^3 - 2}{2x^2}$, $x > 0$ find a) the second derivative;

b) an antiderivative.

Simplify first! [6]

3. Solve $y' = \sin(\frac{x}{3} + \pi)$, $y(\pi) = 0$. [4]

Solution For1)

an equation is $y = m(x-1) - 2$ for m use the implicit differentiation

$(xy^2)' - (y)' = 4\left(\dfrac{x^3}{y}\right)' + 0$ product and quotient rules

$y^2 + x2yy' - y' = 4 \cdot \dfrac{3x^2y - x^3y'}{y^2}$ now $x = 1, y = -2, y' = m$

$4 - 4m - m = 4 \cdot \dfrac{-6-m}{4}$ $\quad 4 - 5m = -6 - m \quad 10 = 4m$

so $m = \dfrac{5}{2}$ and an equation is $y = \frac{5}{2}(x-1) - 2$ or $5x - 2y = 9$

For 2)

$$f(x) = \frac{4\sqrt{x} + x^3 - 2}{2x^2} = \frac{4\sqrt{x}}{2x^2} + \frac{x^3}{2x^2} - \frac{2}{2x^2} = 2x^{-\frac{3}{2}} + \tfrac{1}{2}x - x^{-2}$$

for a)

$f'(x) = 2\left(-\frac{3}{2}\right)x^{-\frac{5}{2}} + \frac{1}{2} - (-2)\,x^{-3} = -3x^{-\frac{5}{2}} + \frac{1}{2} + 2x^{-3}$

$f''(x) = -3\left(-\frac{5}{2}\right)x^{-\frac{7}{2}} + 0 + 2\,(-3)\,x^{-4} = \frac{15}{2}x^{-\frac{7}{2}} - 6x^{-4}, x > 0;$

for b)

$$\int \frac{4\sqrt{x} + x^3 - 2}{2x^2}dx = 2\int x^{-\frac{3}{2}}dx + \tfrac{1}{2}\int xdx - \int x^{-2}dx = 2 \cdot \frac{x^{-\frac{1}{2}}}{\frac{-1}{2}} + \tfrac{1}{2} \cdot \tfrac{x^2}{2} - \tfrac{x^{-1}}{-1} + c =$$

$$= -4x^{-\frac{1}{2}} + \frac{1}{4}x^2 + x^{-1} + c, x > 0.$$

For 3)

$y = \int y'dx = \int \sin(\frac{x}{3} + \pi)dx = -3\cos\left(\frac{x}{3} + \pi\right) + c$

using $\int \sin(ax+b)dx = \frac{-1}{a}\cos(ax+b) + c$ where $a = \frac{1}{3}, b = \pi$

now, $y = 0, x = \pi, c = ?$

using $cos(\theta + \pi) = -\cos\theta$ or $\cos\frac{4}{3}\pi = -\frac{1}{2}$

$0 = -3\cos\left(\frac{\pi}{3} + \pi\right) + c = 3\cos\left(\frac{\pi}{3}\right) + c = \frac{3}{2} + c$ and $c = -\frac{3}{2}$ finally,

$y = -3\cos\left(\frac{x}{3} + \pi\right) - \frac{3}{2}$

ALSO, using $\sin(\frac{x}{3} + \pi) = -\sin\frac{x}{3}$

$y = -\int \sin(\frac{x}{3})dx = 3\cos\frac{x}{3} + c$

and $y = 3\cos\frac{x}{3} - \frac{3}{2}$ for any x.