The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz # 5R Fall 2007

Name:____________________________**I.D.#:**_________________

1. Solve for x: $\log_4 x = \log_{16}(3x-5)$. [5]]
2. Find $f'(x)$ if $f(x) = x^{\cos x} + \ln\sqrt{x} + 3^{\frac{1}{x}}$ for $x > 0$. [5]
3. How much money do you have to invest to get $10,000 in 6 years if the annual interest of 3% is compounded monthly ? [5]

SOLUTION

FOR 1)

$\log_4 x = \dfrac{\ln x}{\ln 4}$ for $x > 0$ and

$\log_{16}(3x-5) = \dfrac{\ln(3x-5)}{\ln 4^2} = \dfrac{\ln(3x-5)}{2\ln 4}$ for $3x-5>0, x > \frac{5}{3}$

so $\dfrac{\ln x}{\ln 4} = \dfrac{\ln(3x-5)}{2\ln 4}$ $\quad 2\ln x = \ln(3x-5)$ $\quad \ln x^2 = \ln(3x-5)$

apply exp.f. $\quad x^2 = 3x-5 \quad x^2 - 3x + 5 = 0$

since the discriminant $D = 9 - 20 = -11 < 0$!!!there is **NO solution**

FOR 2)

express $x^{\cos x} = e^{\cos x \ln x}$ and $\quad \ln\sqrt{x} = \frac{1}{2}\ln x$ $\quad$ for $x > 0$

$$f'(x) = \left(e^{\cos x \ln x}\right)' + (\tfrac{1}{2}\ln x)' + \left(3^{\frac{1}{x}}\right)' = e^{\cos x \ln x}(\cos x \ln x)' + \tfrac{1}{2}(\ln x)' + 3^{\frac{1}{x}}(x^{-1})' =$$

$$= e^{\cos x \ln x}(-\sin x \cdot \ln x + \frac{\cos x}{x}) + \frac{1}{2x} - \frac{3^{\frac{1}{x}}}{x^2} \text{ for } x > 0$$

OR $(\ln\sqrt{x})' = \frac{1}{\sqrt{x}}(\sqrt{x})' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \dfrac{1}{2x}$

you may use log.diff but only for the first part:

$u = x^{\cos x} \quad \ln u = \ln x^{\cos x} = \cos x \cdot \ln x \quad \frac{u'}{u} = (-\sin x \cdot \ln x + \dfrac{\cos x}{x})$ as above

and finally $\quad u' = x^{\cos x}(-\sin x \cdot \ln x + \dfrac{\cos x}{x})$

FOR 3)

the correct formula is $\quad A(t) = A_0\left(1 + \dfrac{p}{100n}\right)^{nt}$ where $p = 3, n = 12, t = 6$ and $A = 10\ 000$

find A_0

so $\quad 10\ 000 = A_0\left(1 + \dfrac{3}{1200}\right)^{12\cdot 6} = A_0\left(\dfrac{401}{400}\right)^{72}$ thus $A_0 = 10\ 000 \cdot \left(\dfrac{400}{401}\right)^{72} = \$\ 8$ 354.58