The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01
Quiz #5W Fall 2007

Name:______________________________**I.D.#:**____________________

1. Solve for x : $\dfrac{1}{3^{2x+1}} = \dfrac{5}{9^{3x}}$. [5]
2. For $f(x) = x^{\frac{1}{x}} + \ln(3x) + 2^{x^2}$ find the domain and $f'(x)$. [5]
3. After 4 days a sample of radon decayed to 35% of its original amount. What is half-life of radon ? [5]

SOLUTION

For 1)

cross multiply first $9^{3x} = 5 \cdot 3^{2x+1}$, then apply ln to both sides

thus

$3x \ln 9 = \ln 5 + (2x+1)\ln 3 \qquad 3x \ln 9 - 2x \ln 3 = \ln 5 + \ln 3 = \ln(5 \cdot 3)$

$3x \ln 3^2 - 2x \ln 3 = (6x - 2x) \ln 3 = 4x \ln 3 = \ln 15$

or $\quad x(3 \ln 9 - 2 \ln 3) = x \ln \frac{9^3}{3^2} = x \ln 9^2 = 4x \ln 3$

and finally $\quad x = \dfrac{\ln 15}{4 \ln 3}$

OR $\quad \dfrac{9^{3x}}{3^{2x+1}} = \dfrac{3^{6x}}{3^{2x+1}} = 3^{4x-1} = 5 \qquad 3^{4x} = 15$

and then log.f. $\quad 4x = \log_3 15 \qquad x = \frac{1}{4}\dfrac{\ln 15}{\ln 3}$

For 2)

for $x > 0$ $\quad$ rewrite $x^{\frac{1}{x}} = e^{\frac{1}{x}\ln x}$

$$f'(x) = \left(e^{\frac{1}{x}\ln x}\right)' + (\ln 2x)' + \left(2^{x^2}\right)' = e^{\frac{1}{x}\ln x}\left(\tfrac{\ln x}{x}\right)' + \frac{1}{2x} \cdot 2 + 2^{x^2}(\ln 2)(x^2)' =$$

$$= e^{\frac{1}{x}\ln x}\left(\frac{x \cdot \frac{1}{x} - \ln x}{x}\right)' + \frac{1}{x} + 2^{x^2}(\ln 2)(2x) = e^{\frac{1}{x}\ln x}\left(\frac{1 - \ln x}{x}\right) + \frac{1}{x} + 2^{x^2}(\ln 2)(2x)$$

also you may simplify$[\ln(2x)]' = (\ln 2 + \ln x)' = 0 + \frac{1}{x}$

You may use log.diif but only for the last part

$u = x^{\frac{1}{x}} \qquad \ln u = \ln x^{\frac{1}{x}} = \frac{1}{x}\ln x \qquad \frac{u'}{u} = \left(\frac{\ln x}{x}\right)' = \dfrac{1 - \ln x}{x}$ as above

and finally $\quad u' = x^{\frac{1}{x}} \cdot \dfrac{1 - \ln x}{x}$

For 3)

the correct formula $A(t) = A_0 e^{kt}$ where $k < 0, t$ in days, $A_0 = 100\%$

first info if $t = 4$ $35 = 100e^{4k}$ solve for k $\ln \frac{35}{100} = 4k$

$k = \frac{\ln 0.35}{4} = -0.2624555$

so $A(t) = 100e^{kt}$ for k calculated above ;now half-life T means

$50 = 100e^{kT}$ or $\frac{1}{2} = e^{kT}$

solve for T $\ln 0.5 = kT$ $T = \frac{4 \ln 0.5}{\ln 0.35} = 2.641$ days