Practice Problems S1

1. Factor the following quadratic expressions:
   (a) $x^2 - 3x - 10$
   (b) $2x^2 + x - 6$
   (c) $4x^2 - 9$.
2. Solve the following inequalities:
   (a) $2x^2 + x < 6$
   (b) $3 - 2x < |3x - 2|$
   (c) $\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1}$
   (d) $|2x + 5| < 11$
   (e) $\frac{1}{|x-1|} < 2$
   (f) $\frac{(3x-2)(x+1)^2}{x(x-1)^3} \leq 0$
3. Find the equation of the line which passes through the point $P(3, 2)$ ans is
   (a) parallel,
   (b) perpendicular

   to the line $2y + 3x + 6 = 0$.
4. Find the centre and radius of the circle $x^2+y^2+3x+7y = \dfrac{3}{2}$ and identify the region represented by the inequality $x^2 + y^2 + 3x + 7y \leq \dfrac{3}{2}$.

Solutions

1. (a) $x^2 - 3x - 10 = (x-5)(x+2)$,
   (b) $2x^2 + x - 6 = 2(x+2)(x - \frac{3}{2})$,
   (c) $4x^2 - 9 = (2x-3)(2x+3)$.

2. (a) The inequality $2x^2 + x < 6$ is equivalent $2x^2 + x - 6 < 0$ or $2(x+2)(x - \frac{3}{2}) < 0$. Therefore, $x \in S = (-2, \frac{3}{2})$.
   (b) For $3 - 2x < |3x - 2|$,

   $$|3x-2| = \begin{cases} 3x - 2 & \text{if} \quad 3x - 2 \geq 0 \Longleftrightarrow x \in [\frac{2}{3}, +\infty) \\ -3x + 2 & \text{if} \quad 3x - 2 < 0 \Longleftrightarrow x \in (-\infty, \frac{2}{3}) \end{cases}.$$

   Case 1: If $x \geq \frac{2}{3}$, then $|3x-2| = 3x - 2$. We have $3 - 2x < 3x - 2 \Longleftrightarrow 5 < 5x \Longleftrightarrow 1 < 0 \Longleftrightarrow x \in (1, \infty)$. So, $x \in S_1 = [\frac{2}{3}, +\infty) \cap (1, \infty) = (1, +\infty)$.
   Case 2: If $x < \frac{2}{3}$, then $|3x-2| = 3x - 2$. We have $3 - 2x < -3x + 2 \Longleftrightarrow x < -1 \Longleftrightarrow x \in (-\infty, -1)$. So, $x \in (-\infty, \frac{2}{3}) \cap (-\infty, -1) = (-\infty, -1)$.
   Finally, the solution set of the inequality is $S = S_1 \cup S_2 = (1, +\infty) \cup (-\infty, -1)$.
   (c) The inequality $\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1}$ is defined for $x \in \mathbb{R} \setminus$

$\{1, -1\}$. For $x \in \mathbb{R} \setminus \{1, -1\}$, we have

$$\begin{aligned} & \frac{2x-1}{x+1} \geq \frac{2x+1}{x-1} \\ \iff & \frac{2x-1}{x+1} - \frac{2x+1}{x-1} \geq 0 \\ \iff & \frac{(2x-1)(x-1) - (2x+1)(x+1)}{(x+1)(x-1)} \geq 0 \\ \iff & \frac{2x^2 - 2x - x + 1 - (2x^2 + 2x + x + 1)}{(x+1)(x-1)} \geq 0 \\ \iff & \frac{-6x}{(x+1)(x-1)} \geq 0 \iff \frac{6x}{(x+1)(x-1)} \leq 0. \end{aligned}$$

Therefore, $S = (-\infty, -1) \cup [0, 1)$.

(d) $|2x+5| < 11 \iff -11 < 2x+5 < 11 \iff -11-5 < 2x < 11-5 \iff -16 < 2x < 6 \iff -8 < x < 3 \iff x \in (-8, 3)$.

(e) The inequality $\frac{1}{|x-1|} < 2$ is defined for all x such that $|x-1| \neq 0$, i.e., $x \neq 1$. For $x \neq 1$, it is equivalent to $\frac{1}{2} < |x-1| \iff x-1 < -\frac{1}{2}$ or $x-1 > \frac{1}{2}$, i.e., $x < \frac{1}{2}$ or $x > \frac{3}{2}$. Therefore, $S = (-\infty, \frac{1}{2}) \cup (\frac{3}{2}, +\infty)$.

(f) This inequality is defined on $\mathbb{R} \setminus \{0, 1\}$ and has solution set $S = (-\infty, 0) \cap (1, \frac{3}{2}]$.

3. (a) The line through $(3, 2)$ (parallel to the line) has equation $y - 2 = -\frac{3}{2}(x-3)$ or $y = -\frac{3}{2}x + \frac{13}{2}$.

(b) Any perpendicular line to $2y+3x+6 = 0$ has slope $m' = -1/m = \frac{2}{3}$. So, $y - 2 = 2/3(x-3) \iff y = \frac{2}{3}x$ is the equation of the line through $(3, 2)$ and perpendicular to $2y + 3x + 6 = 0$.

4. $x^2 + y^2 + 3x + 7y = \frac{3}{2} \iff (x + \frac{3}{2})^2 + (y + \frac{7}{2})^2 - \frac{9}{4} - \frac{49}{4} = \frac{3}{2} \iff (x + \frac{3}{2})^2 + (y + \frac{7}{2})^2 = 16$. It is an equation of the circle centered at $(-\frac{3}{2}, -\frac{7}{2})$ with radius 4. The inequality $x^2+y^2+3x+7y = \frac{3}{2} \iff (x+\frac{3}{2})^2+(y+\frac{7}{2})^2 \leq 16$ represents the closed disk (all points inside the circle and the circle itself).