Practice Problems S3

1. For which values of k are the following functions continuous at $x = a$:

   (a) $f(x) = \begin{cases} 8 - x^2, & x \geq -3 \\ \frac{k}{x}, & x < -3 \end{cases}$ at $x = a = -3$;

   (b) $f(x) = \begin{cases} \frac{\sin(3x)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ at $x = a = 0$.

2. Evaluate the following limits:

   (a) $\lim_{x \to +\infty} \frac{\ln(2x)}{\ln(3x)}$,

   (b) $\lim_{x \to +\infty} \frac{1-e^x}{1+e^x}$,

   (c) $\lim_{x \to 0} \frac{x^2}{1-\cos(x)}$,

   (d) $\lim_{x \to 0} \frac{1-\cos(3x)}{1-\cos(5x)}$,

   (e) $\lim_{x \to 0} \frac{\tan(3x)}{\sin(5x)}$.

3. Find an interval of length

   (a) 1, (b) $\frac{1}{2}$, which contains a root of the equation $x^3 + 4x - 7 = 0$.

4. Use the Intermediate-Value Theorem to prove that the equation $x^3 - 15x + 1 = 0$ has at least two roots in the interval $[0, 4]$.

5. Use the definition of the derivative (i.e., as a limit) to differentiate the following functions:
   (a) $f(x) = \frac{1}{\sqrt{2x+1}}$; (b) $f(x) = \frac{1}{x^2}$.

6. Find the equation of the tangent line to the graph of the function $y = f(x) = \frac{1}{\sqrt{2x+1}}$ at $x = 0$.

7. Given

$$f(x) = \begin{cases} x^3 + 1, & x \geq -1 \\ -3x^2 - 3x, & x < -1 \end{cases}.$$

(a) Find $f(-1)$;

(b) Find $\lim\limits_{h \to 0^-} \frac{f(-1+h) - f(-1)}{h}$ and $\lim\limits_{h \to 0^+} \frac{f(-1+h) - f(-1)}{h}$;

(c) Is $f(x)$ differentiable at $x = -1$? Is $f(x)$ continuous?