

Practice Problems S5

1. Find the derivatives of the following functions:

(a) $f(x) = \frac{1}{\ln(x+e^{-2x})}$;

(b) $f(x) = \log_x(2x + 3)$;

(c) $f(x) = e^{x^2+2x+3} \ln(x + 4)$.

2. Use logarithmic differentiation to find the derivatives of the following functions:

(a) $f(x) = \frac{\sqrt{1+x}(1-x)^{\frac{1}{3}}}{(1+5x)^{\frac{4}{5}}}$;

(b) $f(x) = x^{\cos x}$;

(c) $f(x) = (\sec(x))^x$.

3. Evaluate the following limits using L'Hôpital's rule if possible:

(a) $\lim_{x \rightarrow 0} \frac{xe^x}{1-e^x}$;

(b) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$;

(c) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$;

(d) $\lim_{x \rightarrow 0} \frac{(1-x)e^x - 1}{x \sin x}$;

(e) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$.

4. Find all intervals where the following functions are increasing or decreasing, concave up or concave down:

(a) $f(x) = x^3 + 3x^2 + 1$;

(b) $f(x) = e^{-x^2/2}$.

Does $f(x)$ have inflection points? Explain your answer.