Practice Problems S6

1. Given

$$f(x) = \frac{x^2-9}{x^2-4}, \quad f'(x) = \frac{10x}{(x-2)^2(x+2)^2},$$
$$f''(x) = \frac{-10(3x^2+4)}{(x-2)^3(x+2)^3}, \quad \lim_{x\to-\infty} f(x) = 1 = \lim_{x\to+\infty} f(x),$$
$$\lim_{x\to-2^-} f(x) = -\infty = \lim_{x\to2^+} f(x), \quad \lim_{x\to-2^+} f(x) = +\infty = \lim_{x\to2^-} f(x),$$

sketch the graph of f (Highlight all asymptotes, intercepts, relative extrema and inflection points if any).

2. Let $f(x) = (x^2+x)^{\frac{2}{3}}$.

   (a) Find all critical and singular points of $f(x)$;

   (b) Find the absolute maximum and the absolute minimum values of $f(x)$ on $[-2, 3]$. Where does $f(x)$ attain these values (absolute minimum and absolute maximum points)?

3. Let $f(x) = x^3 - 3x^2 - 9x$.

   (a) Find all relative (local) extrema of $f(x)$ if any;

   (b) Find all intervals where $f(x)$ is concave up or concave down;

   (c) Find all inflection points if any.

4. A cylindrical can, open at the top, is to hold 500 cm^3 of beer. Find the height and the radius that minimize the amount of material needed to manufacture the can.

5. An open box is to be made from a 3 cm by 8 cm rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sizes. Find the maximum volume that the open box can have.

6. If the position S of a particle moving along an s-axis is given as a function of the time t by $S(t) = 2t^3 - 9t^2 + 12t$ for $t > 0$,

   (a) find the velocity, $v(t)$ and acceleration, $a(t)$ of the particle;

   (b) find the average velocity, v_{av} of the particle over the time interval $t_1 = 1$ and $t_2 = 2$.

   (c) find all time intervals when the particle moves in the positive direction and when it moves in the negative direction. When is it stopped?

   (d) find all time intervals for $t > 0$ when the particle is speeding up and when it is slowing down.