MATH 249- 01

Midterm 60 minutes

Fall 2008

1. Calculate

   (a) $\lim_{x\to 2}\frac{\sin(2-x)}{x^2-4} = \lim_{x\to 2}\frac{\sin(2-x)}{(x-2)(x+2)} = \lim_{x\to 2}\frac{\sin(2-x)}{2-x}\cdot\lim_{x\to 2}\frac{-1}{x+2} = 1\cdot\frac{-1}{4} = -\frac{1}{4}$

   using $\lim_{h\to 0}\frac{\sin\ h}{h} = 1$ where $h = 2-x$

   (b) $\lim_{x\to +\infty}\frac{\sin(2-x)}{x^2-4} = 0$ by Squ.Th

   since. $-1 \le \sin(2-x) \le 1$ $\frac{-1}{x^2-4} \le \frac{\sin(2-x)}{x^2-4} \le \frac{1}{x^2-4}$

   and

   both $\lim_{x\to +\infty}\frac{\pm 1}{x^2-4} = 0$

2. $\cos\theta = \frac{1}{3}$ since $\pi < \theta < 2\pi$ $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$

   and then $\sin(2\theta) = 2\sin\theta\cos\theta = 2\cdot\left(-\frac{2\sqrt{2}}{3}\right)\cdot\frac{1}{3} = -\frac{4\sqrt{2}}{9}$

   $\cos(2\theta) = (\cos\theta)^2 - (\sin\theta)^2 = \frac{1}{9} - \frac{8}{9} = -\frac{7}{9}$ and $\cot(2\theta) = \frac{\cos 2\theta}{\sin 2\theta} = -\frac{7}{9}\cdot\frac{9}{-4\sqrt{2}} = \frac{7}{4\sqrt{2}} = \frac{7\sqrt{2}}{8}$

3. For $x = \frac{1}{2}$ $y = f(\frac{1}{2}) = \sqrt{1}\sin\left(\frac{\pi}{2}\right) = 1$ $P\left(\frac{1}{2}, 1\right)$. and a line through P

   is $y = m\left(x - \frac{1}{2}\right) + 1$ for the slope by Product and Chain Rules:

$y' = \left(\sqrt{2x}\right)' \sin \pi x + \sqrt{2x}\,(\sin \pi x)' = \frac{1}{2}\,(2x)^{-\frac{1}{2}} \cdot (2x)' \sin \pi x + \sqrt{2x} \cos \pi x \cdot (\pi x)' =$

$= \frac{\sin \pi x}{\sqrt{2x}} + \pi \sqrt{2x} \cos \pi x$ also $\left(\sqrt{2x}\right)' = \left(\sqrt{2}x^{\frac{1}{2}}\right)' = \sqrt{2} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{x}}$

at $x = \frac{1}{2}$ $m = \sin \frac{\pi}{2} + \pi \cos \frac{\pi}{2} = 1 = 0 = 1$ and $y = x - \frac{1}{2} + 1 = x + \frac{1}{2}$.

4. the graph of $y = p(x) = 10x^3 - 3x^2 + 15$ may cross the $x-$ axis 3times but must at least once

   to decide find Critical Points first: $p'(x) = 30x^2 - 6x = 6x\,(5x-1) = 0$ $x = 0, \frac{1}{5}$

   calculate the y-coordinates: $x = 0 \rightarrow y = 15$ $x = \frac{1}{5} \rightarrow y = \frac{10}{5^3} - \frac{3}{5^2} + 15 = 15 + \frac{1-3}{5^2} = 14\frac{23}{25} > 0$

   thus the graph crosses the x-axis only once on the left of 0

   check some values $p(-1) = -10 - 3 + 15 = 2 > 0, p(-2) = -10 \cdot 8 - 3 \cdot 4 + 15 = -77$

   since p is cont. by IVT applied to $[a, b]$ where $a = -2$ $p(a) < 0$ $b = -1$ $p(b) > 0$

   ther must be a root c between a and b $c \in (-2, -1)$ where $p(c) = 0$.

5. the function f is continuous everywhere except $x = -2, 3$

   for $x = -2:$

   $f(-2) = 4a + b$ $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (ax^2 + b) = 4a + b$

   and $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} 6 \cos \frac{6\pi}{x} = 6 \cos(-3\pi) = -6$

together $\quad 4a + b = -6$

for $\; x = 3:$

$f(3) = 9a + b \qquad \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax^2 + b) = 9a + b$

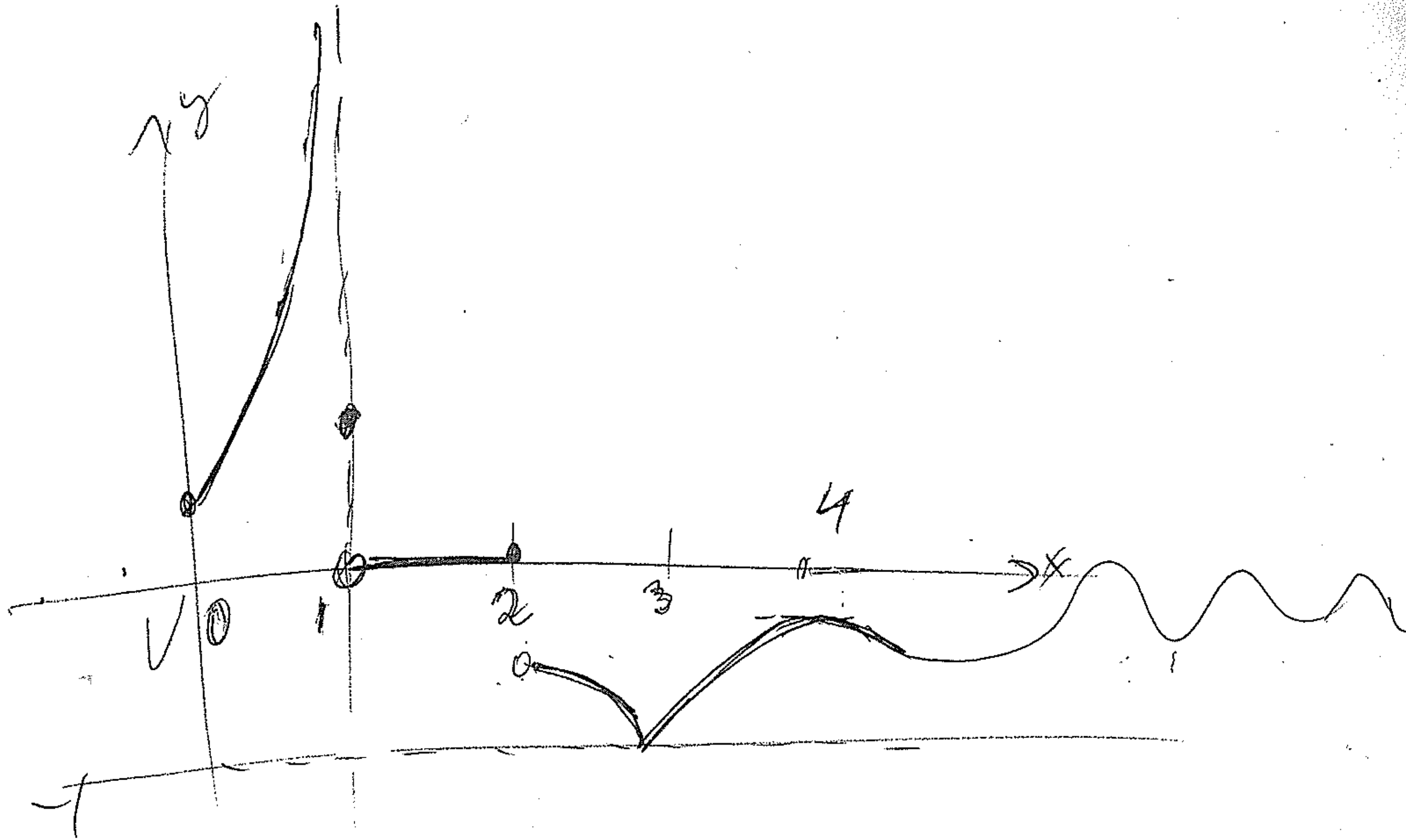
and $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \sin\left(\frac{\pi}{2}x\right) = \sin\frac{3\pi}{2} = -1$

together $\quad 9a + b = -1$

solve the sytem: $\quad 4a + b = -6 \qquad 9a + b = -1$

subtract to get $\quad 5a = -1 + 6 = 5 \to a = 1$ and $b = -1 - 9a = -10$

6.


y
x
0
1
2
3
4
-1
1