

**MATH 249**  
**Midterm Handout**

1. Evaluate

(a)  $\lim_{x \rightarrow \infty} (x^2 - x^2 \cos \frac{1}{x})$       (b)  $\lim_{x \rightarrow 0} (x^2 - x^2 \cos \frac{1}{x})$

2. Evaluate

(a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x - \pi}$       (b)  $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi}$       (c)  $\lim_{x \rightarrow -\infty} \frac{\sin x}{x - \pi}$ .

3. For  $y = \frac{\cos \pi x}{1 - x}$  find an equation of the tangent line at  $x = -\frac{1}{2}$ .

4. For  $y = \left(\sin \frac{1}{\sqrt{x^4+1}}\right)^3$  find  $y'$ .

5. Show that the function  $f(x) = x - 2 \sin(\pi x)$  has at least one positive zero i.e.  $f(x) = 0$  at least for one  $x > 0$ .

6. Locate all 3 roots of  $p(x) = 2x^3 - 6x^2 + 7$  i.e. find 3 intervals each containing one root. Sketch the graph of  $y = p(x)$ .

7. Find  $\sec \theta$  if  $\sin \theta = \frac{1}{5}$  and  $\frac{\pi}{2} < \theta < \frac{3}{2}\pi$ . NO calculator.

8. If  $\cos \theta = \frac{2}{3}$  and  $\pi < \theta < 2\pi$  find  $\sin \theta$  and then  $\sin 2\theta$ . No calculator.

9. Find the values of  $a$  and  $b$  so that the function  $f$  is continuous everywhere

$$f(x) = \begin{cases} \left(\frac{2}{2x+1} - 3\right)(4x^2 - 1) & \text{for } x < -\frac{1}{2} \\ ax + b & \text{for } -\frac{1}{2} \leq x \leq 2 \\ \cos\left(-\frac{\pi}{x}\right) & \text{for } x > 2 \end{cases} .$$

10. Find the values of  $a$  and  $b$  so that the function  $f$  is continuous everywhere

$$f(x) = \begin{cases} \cos(\pi x) - 2 \sin \frac{\pi x}{2} & \text{for } x > 3 \\ ax^2 + b & \text{for } 0 \leq x \leq 3 \\ 6 \cdot \frac{\sqrt{9-x} - 3}{x} & \text{for } x < 0 \end{cases} .$$

11. **A**

Sketch the graph of ONE function satisfying all the following conditions:

(a)  $f$  is defined on  $[-2, +\infty)$

(b)  $f$  is discontinuous at  $x = 0, 1, 2$  where  $\lim_{x \rightarrow 1} f(x) = 3$ ,  $\lim_{x \rightarrow 2} f(x)$  DNE (does not exist). otherwise continuous

(c)  $x = 0$  is a vertical asymptote and  $y = -2$  is a horizontal asymptote

(d)  $f$  is not differentiable at  $x = -1, 0, 1, 2$  (no  $f'(-1)$ ) otherwise differentiable and  $f'(x) = 0$  for all  $x \in (0, 1)$ , also  $f'(4) = 0$ .

- (e)  $f$  is increasing on  $(-2, -1)$  and on  $(2, 4)$ ; decreasing on  $(-1, 0)$  and on  $(4, +\infty)$ ;
- (f) the maximum value is 3.

### B

Sketch the graph of ONE function satisfying all the following conditions:

- (a)  $f$  is defined on  $(-\infty, 1]$
- (b)  $f$  is discontinuous at  $x = -3$  and  $x = -2$  where  $\lim_{x \rightarrow -3^+} f(x) = f(-3) = 5$  otherwise continuous
- (c)  $x = -2$  is a vertical asymptote and  $\lim_{x \rightarrow -\infty} f(x) DNE$  (does not exist)
- (d)  $f$  is not differentiable at  $x = -1, -2, -3$  (no  $f'(-1)$ ) otherwise differentiable and  $f'(x) = 0$  for all  $x \in (-1, 0)$ , also  $f'(-4) = 0$ ;
- (e)  $f$  is increasing on  $(-3, -2)$  and on  $(0, 1)$ ; decreasing on  $(-2, -1)$ ;
- (f) the minimum value is  $-2$ .

### C

Sketch the graph of ONE function satisfying all the following conditions:

- (a)  $f$  is defined on  $(-\infty, 2)$
- (b)  $f$  is discontinuous at  $x = -3$  and  $x = -2$  where  $\lim_{x \rightarrow -3} f(x) = 2$ , and  $x = -2$  is a vertical asymptote, otherwise continuous
- (c)  $y = 1$  is a horizontal asymptote
- (d)  $f$  is not differentiable at  $x = -1, -2, -3$  (no  $f'(-1)$ ) otherwise differentiable and  $f'(x) = 0$  for all  $x \in (1, 2)$ , also  $f'(-4) = 0$ ;
- (e)  $f$  is increasing on  $(-1, 1)$  and on  $(-3, -2)$ ; decreasing on  $(-2, -1)$ ;
- (f) the minimum value is  $\frac{1}{2}$ .

### D

Sketch the graph of ONE function satisfying all the following conditions:

- (a)  $f$  is defined on  $(-1, \infty)$
- (b)  $f$  is discontinuous at  $x = 3$  and  $x = 2$  where  $\lim_{x \rightarrow 2^+} f(x) = f(2) = 3$ ,  $x = 3$  is a vertical asymptote, otherwise continuous
- (c) and  $\lim_{x \rightarrow +\infty} f(x) DNE$  (does not exist)
- (d)  $f$  is not differentiable at  $x = 0, 2, 3$  (no  $f'(0)$ ) otherwise differentiable and  $f'(x) = 0$  for all  $x \in (-1, 0)$ , also  $f'(4) = 0$ .
- (e)  $f$  is increasing on  $(2, 3)$  and on  $(3, 4)$ ; decreasing on  $(0, 2)$ ;

**E**

Sketch the graph of ONE function satisfying all the following conditions:

- (a)  $f$  is defined on  $(-\infty, +\infty)$
- (b)  $f$  is discontinuous at  $x = -1$  and  $x = 2$  where  $\lim_{x \rightarrow -1} f(x)$  DNE  
 $x = 2$  is a vertical asymptote ,otherwise continuous
- (c) and  $\lim_{x \rightarrow -\infty} f(x)$  DNE (does not exists),  $y = -3$  is a horizontal asymptote;
- (d)  $f$  is not differentiable at  $x = -1, 1, 2$  (no  $f'(1)$ ) otherwise differentiable  
and  $f'(x) = 0$  for all  $x \in (2, 3, )$ also  $f'(4) = 0$ ;
- (e)  $f$  is increasing on  $(-1, 1)$  ; decreasing on  $(1, 2)$  and on  $(4, +\infty)$
- (f) the maximum value is 4.