The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 2R

Fall,2008

Name:______________________________**I.D.#:**__________________

1. For $f(x) = \sqrt{x+3}$ and $g(x) = \dfrac{3}{x+3}$ find $f \circ g$,$g \circ g$ and thier domains. [4]

2. For $\quad f(x) = \dfrac{|x-2| - x + 2}{x^2 - x - 2} \quad$ find $\lim f(x)$ as

   (a) (a) $x \to 2^-$ (b) $x \to +\infty$ (c) $x \to -1^+$ [3]

3. For $\quad g(x) = \dfrac{1}{x-2}\left(x - \dfrac{4}{x}\right) \quad$ find $\lim g(x)$ as

   (a) (a) $x \to 2$ (b) $x \to 0^-$ (c) $x \to +\infty$ [3]

SOLUTION

For 1)

domains $\quad D_f = \{x \geq -3\} = [-3, +\infty) \quad D_g = \{x \neq -3\}$

$(f \circ g)(x) = f(g(x)) = \sqrt{(..)+3} = \sqrt{\dfrac{3}{x+3} + 3} =$

we can simplify $\quad \sqrt{\dfrac{3+3x+9}{x+3}} = \sqrt{\dfrac{3x+12}{x+3}}$

for the domain solve $\quad \dfrac{3(x+4)}{x+3} \geq 0$

split points $x = -4, -3$ are switch points

testing$- -^{pos} - -_{-4} -^{neg} - -_{-3} -^{pos} --$

therefore $\quad D_{f \circ g} = (-\infty, -4] \cup (-3, \infty)$

for $x \neq -3$

$$(g \circ g)(x) = g(...) = \frac{3}{\left(\dfrac{3}{x+3}\right) + 3} = \frac{3}{\dfrac{3+3x+9}{x+3}} = \frac{3\,(x+3)}{3\,(x+4)} = \frac{x+3}{x+4}, x \neq -4$$

$D_{g \circ g} = \{x \neq -4, -3\}$

For 2a)

since $x < 2 \quad x - 2 < 0 \quad |x-2| = -(x-2) = 2 - x$

$$.f(x) = \frac{|x-2| - x + 2}{x^2 - x - 2} = \frac{2 - x - x + 2}{(x-2)(x+1)} = \frac{2(2-x)}{(x-2)(x+1)} = \frac{-2}{x+1}$$

$\lim_{x \to 2^{--}} f(x) = \frac{-2}{3}$

For 2b)

as $x \to +\infty \qquad x > 2$ and $x - 2 > 0 \qquad |x-2| = x - 2$

$f(x) = \frac{|x-2| - x + 2}{x^2 - x - 2} = \frac{x - 2 - x + 2}{x^2 - x - 2} = 0$ so $\lim_{x \to +\infty} f(x) = 0$

For 2c)

since $x > -1 \qquad x + 1 > 0$

and x is close to 1 $\qquad x - 2 < 0 \qquad |x-2| = -(x-2) = 2 - x$

$.f(x) = \frac{|x-2| - x + 2}{x^2 - x - 2} = \frac{2 - x - x + 2}{(x-2)(x+1)} = \frac{2(2-x)}{(x-2)(x+1)} = \frac{-2}{x+1}$

$\lim_{x \to -1^+} f(x) = \frac{-2}{0^+} = -\infty$

For 3)

simplify first for $x \neq 2, 0$

$$g(x) = \frac{1}{x-2}\left(x - \frac{4}{x}\right) = \frac{1}{x-2}\left(\frac{x^2-4}{x}\right) = \frac{1}{x-2} \cdot \frac{(x-2)(x+2)}{x} = \frac{x+2}{x}$$

in **a**) as $x \to 2$ **limit** is 2; .

in **b**) as $x \to 0^-$ **limit** is "$\frac{2}{0^-}$" $= -\infty$;

in **b**) as $x \to +\infty \qquad \frac{x+2}{x} = 1 + \frac{2}{x} \to 1$

or from the original

$$g(x) = \frac{1}{x-2}\left(x - \frac{4}{x}\right) = \frac{1}{x-2}\left(\frac{x^2-4}{x}\right) = \frac{x^2-4}{x^2-2x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{1 - \frac{4}{x^2}}{1 - \frac{2}{x}} \to 1$$