The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 2W

Fall 2008

Name:______________________________**I.D.#:**__________________

1. For $f(x) = \dfrac{3x^2+5x-2}{4-x^2}$ find $\lim f(x)$

   (a) as $x \to -2$; (b)as $x \to -\infty$; (c) as $x \to 2^-$ [3]

2. For $g(x) = \dfrac{1-x}{2-\sqrt{x+3}}$ find $\lim g(x)$

   (a) as $x \to 1$; (b) as $x \to +\infty$ (c) as $x \to -\infty$. [3]

3. For $f(x) = \dfrac{x}{x-8}$ and $g(x) = \sqrt{1-x}$ find the compositions

   $f \circ g$, $f \circ f$ and their domains. [4]

SOLUTION

For 1a)

first we can simplify for $x \neq -2, 2$

$$f(x) = \frac{3x^2+5x-2}{4-x^2} = \frac{(3x-1)(x+2)}{(2-x)\,(2+x)} = \frac{(3x-1)}{(2-x)}$$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{3x-1}{2-x} = -\frac{7}{4}$$

for c)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 3} \frac{3x-1}{2-x} = \text{"}\frac{5}{0^+}\text{"} = +\infty \text{ since } \; x<2 \qquad 0<2-x$$

for b)

$$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{3x-1}{2-x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to \pm\infty} \frac{3-\frac{1}{x}}{\frac{2}{x}-1} = -3$$

also from the original

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{3x^2+5x-2}{4-x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \text{"} \lim_{x\to -\infty} \frac{3+\frac{5}{x}-\frac{2}{x^2}}{\frac{4}{x^2}-1} = \frac{3}{-1} = -3$$

For 2 a)

g is defined for $x \in [-3, +\infty)$ and $x \neq 1$

$(x+3) \geq 0$

$g(x) = \dfrac{1-x}{2-\sqrt{x+3}} \cdot \dfrac{2+\sqrt{x+3}}{2+\sqrt{x+3}} = \dfrac{(1-x)\left(2+\sqrt{x+3}\right)}{2^2 - \left(\sqrt{x+3}\right)^2} = \dfrac{(1-x)\left(2+\sqrt{x+3}\right)}{4-x-3} =$

$(2+\sqrt{x+3})$

and $\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1}(2+\sqrt{x+3}) = 4$

for b) we can use simplified form

$\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty}(2+\sqrt{x+3}) = +\infty$

OR the original form

$$\lim_{x\to+\infty} \frac{1-x}{2-\sqrt{x+3}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}} = \lim_{x\to+\infty} \frac{\frac{1}{\sqrt{x}} - \sqrt{x}}{\frac{2}{\sqrt{x}} - \frac{\sqrt{x+3}}{\sqrt{x}}} =$$

$$= \lim_{x\to+\infty} \frac{\frac{1}{\sqrt{x}} - \sqrt{x}}{\frac{2}{\sqrt{x}} - \sqrt{1+\frac{3}{x}}} = \frac{-\infty}{-1} = +\infty$$

but **for c)** for $x < -3$ $\quad g(x) = \dfrac{1-x}{2-\sqrt{x+3}}$ is not defined

so

$\lim\limits_{x\to-\infty} g(x) \qquad DNE$ =does NOT exist

For 3)

First domain of the given functions

for $f(x) = \dfrac{x}{x-8} \qquad x-8 \neq 0 \ D_f = \{x \neq 8\}$ and

for $g(x) = \sqrt{1-x}$ it must $1-x \geq 0$,for $x \leq 1 \qquad D_g = (-\infty, 1]$

$(f \circ g)(x) = f(...) = \dfrac{()}{()-8} = \dfrac{\sqrt{1-x}}{\sqrt{1-x}-8}$ and $\sqrt{1-x} - 8 \neq 0, 1-x \neq 64, x \neq -63$

$D_{f\circ g} = (-\infty, -63) \cup (-63, 1]$

for $x \neq 8$

$$(f \circ f)\ (x) = f(...) = \frac{\left(\frac{x}{x-8}\right)}{\left(\frac{x}{x-8}\right) - 8} = \frac{\frac{x}{x-8}}{\frac{x-8(x-8)}{x-8}} = \frac{x\,(x-8)}{(x-8)\,(64-7x)} = \frac{x}{64-7x}$$

$\{x \neq 8 \qquad x \neq \dfrac{64}{7}\} = D_{f\circ f}$