

The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz #3R

FALL 2008

Name: _____ I.D.#: _____]

1. Using the **definition of the derivative** find $f'(-4)$ if $f(x) = \sqrt{1-2x}$. [4]

2. Find $f'(x)$ if $f(x) = \frac{\sqrt{4x}}{x^2+5}$ at $x = 1$. [3]

3. Find all points on the graph $y = \left(\frac{x^2}{3} - 3\right)^3$ with horizontal tangents. [3]

For 1) $f(-4) = \sqrt{1+8} = 3$

$$\begin{aligned} f'(-4) &= \lim_{x \rightarrow -4} \frac{f(x) - f(-4)}{x + 4} = \lim_{x \rightarrow -4} \frac{\sqrt{1-2x} - 3}{x + 4} \cdot \frac{\sqrt{1-2x} + 3}{\sqrt{1-2x} + 3} = \\ &= \lim_{x \rightarrow -4} \frac{1 - 2x - 3^2}{x + 4} \cdot \frac{1}{\sqrt{1-2x} + 3} = \lim_{x \rightarrow -4} \frac{-2(x+4)}{x + 4} \cdot \frac{1}{\sqrt{1-2x} + 3} = \\ &= \lim_{x \rightarrow -4} \frac{-2}{\sqrt{1-2x} + 3} = \frac{-2}{6} = -\frac{1}{3} \end{aligned}$$

OR

$$\begin{aligned} f'(-4) &= \lim_{h \rightarrow 0} \frac{f(-4+h) - f(-4)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1-2(-4+h)} - 3}{h} = \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{9-2h} - 3}{h} \cdot \frac{\sqrt{9-2h} + 3}{\sqrt{9-2h} + 3} = \lim_{h \rightarrow 0} \frac{9-2h-3^2}{h} \cdot \frac{1}{\sqrt{9-2h} + 3} = \\ &= \lim_{h \rightarrow 0} \frac{-2h}{h} \cdot \frac{1}{\sqrt{9-2h} + 3} = \frac{-2}{3+3} = -\frac{1}{3}. \end{aligned}$$

Check by Rules: $f'(x) = \frac{1}{2}(1-2x)^{-\frac{1}{2}} \cdot (-2)$ and at $x = -4$,

$$f'(-4) = \frac{-1}{\sqrt{9}} = -\frac{1}{3}.$$

For 2) Simplify first $\sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x}$

then for $x > 0$ by Quotient Rule

$$\begin{aligned} f'(x) &= \left(\frac{\sqrt{4x}}{x^2+5}\right)' = \left(\frac{2x^{\frac{1}{2}}}{x^2+5}\right)' = \frac{(2x^{\frac{1}{2}})'(x^2+5) - 2\sqrt{x}(x^2+5)'}{(x^2+5)^2} = \\ &= \frac{2 \cdot \frac{1}{2}x^{-\frac{1}{2}}(x^2+5) - 2\sqrt{x} \cdot 2x}{(x^2+5)^2} = \frac{\frac{1}{\sqrt{x}}(x^2+5) - 4x\sqrt{x}}{(x^2+5)^2} \end{aligned}$$

at $x = 1$

$$f'(1) = \frac{6-4}{36} = \frac{1}{18}$$

Or

if not simplified you have to use Chain Rule

$$\begin{aligned}
(\sqrt{4x})' &= \frac{1}{2}(4x)^{-\frac{1}{2}} \cdot (4x)' = \frac{1}{2}(4x)^{-\frac{1}{2}} \cdot 4 = \frac{2}{\sqrt{4x}} \\
f'(x) &= \left(\frac{\sqrt{4x}}{x^2+5} \right)' = \frac{(\sqrt{4x})'(x^2+5) - \sqrt{4x}(x^2+5)'}{(x^2+5)^2} = \\
&= \frac{\frac{2}{\sqrt{4x}}(x^2+5) - \sqrt{4x} \cdot 2x}{(x^2+5)^2} \text{ and at } x=1, f'(1) = \frac{\frac{2}{2} \cdot 6 - 2 \cdot 2}{36} = \frac{1}{18}.
\end{aligned}$$

For 3)

horizontal tangent means $m = y' = 0$, by Chain Rule

$$y' = \left[\left(\frac{x^2}{3} - 3 \right)^3 \right]' = 3 \left(\frac{x^2}{3} - 3 \right)^2 \cdot \left(\frac{1}{3}x^2 - 3 \right)' = 3 \left(\frac{x^2}{3} - 3 \right)^2 \cdot \frac{1}{3} \cdot 2x = 2x \left(\frac{1}{3}x^2 - 3 \right)^2$$

solve for x $y' = 0$

For $x = 0, y = (-3)^3 = -27$, so one point is $P(0, -27)$

for $\frac{x^2}{3} - 3 = 0 \rightarrow x^2 = 9 \quad x = \pm 3, y = \left(\frac{9}{3} - 3 \right)^3 = 0$ two more points $(\pm 3, 0)$.