The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz #3W

FALL 2008

Name:______________________________**I.D.#:**__________________

1. Using the definition of derivative find $f'(-2)$ if $f(x) = \dfrac{x}{3+x}$. [3]

2. Find y' if $y = (\dfrac{x^2}{4} - \dfrac{1}{2\sqrt{x}})\sqrt{2x+3}$ for $x > 0$. [3]

3. Find an equation of the tangent line to $y = x\ \sqrt[3]{x^2+4}$ at $x = 2$. [4]

Solution
For 1)

$$f(-2) = \frac{-2}{1} \qquad f'(-2) = \lim_{x\to -2}\frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2}\frac{\frac{x}{3+x}-(-2)}{x+2} = \lim_{x\to -2}\frac{\frac{x}{3+x}+2}{x+2} =$$

$$= \lim_{x\to -2}\frac{\frac{x+6+2x}{3+x}}{x+2} = \lim_{x\to -2}\frac{\frac{3(x+2)}{3+x}}{x+2} = \lim_{x\to -2}\frac{3(x+2)}{(3+x)(x+2)} = \lim_{x\to -2}\frac{3}{3+x} = 3$$

ALSO

$$f'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0}\frac{\frac{(-2+h)}{3+(-2+h)}-(-2)}{h} =$$

$$= \lim_{h\to 0}\frac{\dfrac{h-2}{1+h}+2}{h} = \lim_{h\to 0}\frac{1}{h}\left[\frac{h-2+2+2h}{1+h}\right] =$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{3h}{1+h} = \lim_{h\to 0}\frac{3}{1+h} = 3$$

(Check by rules $f'(x) = \dfrac{1}{3+x} - \dfrac{x}{(3+x)^2}$ at $x = -2$ $\qquad f'(-2) = 1 + \frac{2}{1} = 3$)

For 2)

use Product Rule $\qquad y' = (\dfrac{x^2}{4} - \dfrac{1}{2\sqrt{x}})'\sqrt{2x+3} + (\dfrac{x^2}{4} - \dfrac{1}{2\sqrt{x}})\left(\sqrt{2x+3}\right)'$

$$y' = (\frac{1}{4}x^2 - \frac{1}{2}x^{-\frac{1}{2}})'\sqrt{2x+3} + (\frac{x^2}{4} - \frac{1}{2\sqrt{x}})\left((2x+3)^{\frac{1}{2}}\right)'$$

now Power and Chain Rules $\qquad y' = (\dfrac{1}{4}\cdot 2x - \dfrac{1}{2}\cdot\dfrac{-1}{2}x^{-\frac{3}{2}})\sqrt{2x+3} + (\dfrac{x^2}{4} - \dfrac{1}{2\sqrt{x}})\cdot \dfrac{1}{2}(2x+3)^{-\frac{1}{2}}(2x+3)'$

and $(2x+3)' = 2$

so $\qquad y' = (\dfrac{1}{2}x + \dfrac{1}{4}x^{-\frac{3}{2}})\sqrt{2x+3} + (\dfrac{x^2}{4} - \dfrac{1}{2\sqrt{x}})(2x+3)^{-\frac{1}{2}}$ for $x > 0$

For 3)

for $x = 2$ $y = 2 \cdot \sqrt[3]{8} = 4$ so $y = m(x-2) + 4$

slope of the tangent is given by $y' = \left(x\sqrt[3]{x^2+4}\right)'$ at $x = 2$

we can use Product and Chain Rules

$y' = (x)'\sqrt[3]{x^2+4} + x\left((x^2+4)^{\frac{1}{3}}\right)' = \sqrt[3]{x^2+4} + x \cdot \frac{1}{3}\left(x^2+4\right)^{-\frac{2}{3}} \cdot 2x = \sqrt[3]{x^2+4} + \frac{2x^2}{3\left(x^2+4\right)^{\frac{2}{3}}}$

at $x = 2$ $\sqrt[3]{x^2+4} = 8^{\frac{1}{3}} = 2$ $(x^2+4)^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 4$ so $m = 2 + \frac{8}{3 \cdot 4} = 2 + \frac{2}{3} = \frac{8}{3}$

and $y = \frac{8}{3}(x-2) + 4$ or $y = \frac{8}{3}x - \frac{4}{3}$ or $8x - 3y = 4$

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