The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 4R

FALL 2006

Name:______________________________**I.D.#:**__________________

1. Find an equation of the tangent to

$$x^2y - y^2 = \frac{2x^3}{y} - 1$$

at the point $(-1, 2)$. [4]

2. Find the second derivative of $f(x) = \cos(3 - x^2)$. [3]

3. Find a general antiderivative of $f(x) = \dfrac{(2x - \frac{3}{\sqrt{x}})}{x}$ for $x > 0$. [3]

Solution

For 1) an equation $y = m\,(x+1) + 2$ and to find m use

implicit differentiation $[x^2y - y^2]' = 2\left(\dfrac{x^3}{y}\right)' - 0$

$(x^2y)' - 2yy' = 2 \cdot \dfrac{(x^3)'y - x^3y'}{y^2}$ multiply by 2

$2xy + x^2y' - 2yy' = 2 \cdot \dfrac{3x^2y - x^3y'}{y^2}$;now $x = -1, y = 2, y' = m$

$-4 + m - 4m = 2 \cdot \dfrac{6+m}{2^2} \to -4 - 3m = \dfrac{6+m}{2}$

so $-8 - 6m = 6 + m$ $\quad -14 = 7m$ $\quad m = -2$

and

$y = -2\,(x+1) + 2$ OR $y = -2x$

For 2) by Chain rule

$f'(x) = [\cos(3 - x^2)]' = -\sin(3 - x^2) \cdot (3 - x^2)' =$

$= -\sin(3 - x^2) \cdot (-2x) = 2x\sin(3 - x^2)$

by Product and Chain Rules

$f''(x) = (2x)'\sin(3 - x^2) + 2x\,[\sin(3 - x^2)]' = 2\sin(3 - x^2) + 2x\cos(3 - x^2) \cdot (-2x) =$

$= 2\sin(3 - x^2) - 4x^2\cos(3 - x^2)$.

For 3) get rid of quotient = simplify first

$\displaystyle\int \frac{(2x - \frac{3}{\sqrt{x}})}{x}dx = \int \frac{(2x - 3x^{-\frac{1}{2}})}{x}dx =$

$\displaystyle =2\int \frac{x}{x}dx - 3\int x^{-\frac{3}{2}}dx$

$\displaystyle = 2\int dx - 3 \cdot \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} = 2x + 6x^{-\frac{1}{2}} + c, x > 0$