**The University of Calgary**
**Department of Mathematics and Statistics**
**MATH 249//01**
**Quiz # 4W**

**FALL 2008**

**Name:_____I.D.#:_____**

1. Find an equation of the tangent line to

$$\sin(xy) - xy^2 = \frac{\pi}{2} - 1$$

at the point $(-\frac{\pi}{2}, 1)$. [3]

2. Find the second derivative of $f(x) = \frac{x}{1+2x^2}$. Simplify. [4]

3. Find a general **antiderivative** of $f(x) = \sqrt{x}\left(4\sqrt{x} - \frac{1}{4x}\right)$ for $x > 0$. [3]

**Solution For1)**

an equation is $y = m(x + \frac{\pi}{2}) + 1$ for $m$ use the implicit differentiation

$\cos(xy) \cdot (xy)' - (xy^2)' = 0$ product and chain rules

$\cos(xy) \cdot (y + xy') - (y^2 + x2yy') = 0$

substitute $x = -\frac{\pi}{2}$ $y = 1$ $y' = m$ $\cos(-\frac{\pi}{2}) = 0$

$-(1 - \pi m) = 0$ $\pi m = 1$ $m = \dfrac{1}{\pi}$

and an equation is $y = \dfrac{1}{\pi}(x + \dfrac{\pi}{2}) + 1$

**For 2)** by Quotient Rule

$$f'(x) = \left(\frac{x}{1+2x^2}\right)' = \frac{1 + 2x^2 - x \cdot 4x}{(1+2x^2)^2} = \frac{1-2x^2}{(1+2x^2)^2}$$

$$f''(x) = \left(\frac{1-2x^2}{(1+2x^2)^2}\right)' = \frac{-4x(1+2x^2)^2 - (1-2x^2)\,2(1+2x^2)4x}{(1+2x^2)^4} =$$

$$= \frac{-4x(1+2x^2)[1+2x^2+2-4x^2]}{(1+2x^2)^4} = \frac{-4x[3-2x^2]}{(1+2x^2)^3}$$

OR by Product and Chain Rules

$$f'(x) = \left[x(1+2x^2)^{-1}\right]' = (1+2x^2)^{-1} - x(1+2x^2)^{-2}\,4x = (1+2x^2)^{-1} - 4x^2(1+2x^2)^{-2}$$

$$f''(x) = -4x(1+2x^2)^{-2} - 8x(1+2x^2)^{-2} - 4x^2(-2)(1+2x^2)^{-3}4x$$

$$= -12x(1+2x^2)^{-2} + 32x^3(1+2x^2)^{-3}.$$

**For 3)** get rid of the product first = simplify

$$\int \sqrt{x}\left(4\sqrt{x} - \frac{1}{4x}\right)dx = 4\int \sqrt{x}\sqrt{x}dx - \frac{1}{4}\int \frac{x^{\frac{1}{2}}}{x}dx =$$

$$= 4\int x dx - \frac{1}{4}\int x^{-\frac{1}{2}}dx = 2x^2 - \frac{1}{2}x^{\frac{1}{2}} + c, x > 0.$$