

The University of Calgary  
 Department of Mathematics and Statistics  
 MATH 249L 03  
 Quiz # 5R

Fall 2008

Name: \_\_\_\_\_ I.D.#: \_\_\_\_\_

1. Evaluate

(a)  $\lim_{x \rightarrow 0} \frac{x}{4^x - 1}$

(b)  $\lim_{x \rightarrow -\infty} \frac{x}{4^x - 1}$  [3]

2. Find  $y'$  if  $y = x^x + \ln(x^2 + 1)$ , for  $x > 0$ . [4]

3. How much money do you have to invest to get \$1000 in 5 years if the annual interest of 2% is compounded monthly? [3]

**SOLUTION**

**For 1a)** since  $4^0 = 1$

$$\lim_{x \rightarrow 0} \frac{x}{4^x - 1} = \frac{0}{0} \text{ (L'H.)} = \lim_{x \rightarrow 0} \frac{(x)'}{(4^x - 1)'} = \lim_{x \rightarrow 0} \frac{1}{4^x \ln 4} = \frac{1}{\ln 4}$$

**for b)**  $\lim_{x \rightarrow -\infty} \frac{x}{4^x - 1} = \frac{-\infty}{-1} = +\infty$  (NO L'H.) since " $4^{-\infty} = 0$ "

**For 2)**

by Chaint Rule and then product Rule

$$\begin{aligned} y' &= e^{x \ln x} (x \ln x)' + [\ln(x^2 + 1)]' = \\ &= e^{x \ln x} \left( \ln x + x \cdot \frac{1}{x} \right) + \frac{1}{x^2 + 1} (x^2 + 1)' = \\ &= e^{x \ln x} (\ln x + 1) + \frac{2x}{x^2 + 1} \end{aligned}$$

also by log.diff.BUT only the first part  $u = x^x$

$$\ln u = \ln [x^x] = x \ln x$$

$$\frac{u'}{u} = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1 \text{ and}$$

$$u' = x^x [\ln x + 1] \text{ then}$$

$$y' = u' + [\ln(x^2 + 1)]' = \dots \text{as above}$$

**For 3)**

the correct formula is  $A(t) = A_0 \left( 1 + \frac{p}{100n} \right)^{nt}$  where  $p = 2, n = 12, t = 5$

and  $A = 1000$   $A_0 = ?$   
 $1000 = A_0 \left( 1 + \frac{2}{1200} \right)^{5 \cdot 12} = A_0 \left( \frac{601}{600} \right)^{60}$  thus  $A_0 = 1000 \cdot \left( \frac{600}{601} \right)^{60} = \$ 9049.127$