The University of Calgary
Department of Mathematics and Statistics
MATH 249L 03
Quiz # 5W

Fall 2008

Name:_____I.D.#:_____

1. Evaluate

   (a) $\lim_{x\to 1}\dfrac{\ln x}{x^2+x-2}$

   (b) $\lim_{x\to 0^+}\dfrac{\ln x}{x^2+x-2}$. [3]

2. Find y' if $y = 3^{x^2} + x^{x^2}$,for $x > 0$. [4]

3. If the half-life of radon-x is 5 years.what percentage of the amoun present now will be remaining after 2 years? [3]

SOLUTION
For 1a)
since $\ln 1 = 0$

$$\lim_{x\to 1}\frac{\ln x}{x^2+x-2} = \text{"}\frac{0}{0}\text{"}(\text{L'H.}) = \lim_{x\to 1}\frac{\frac{1}{x}}{2x+1} = \frac{1}{3}$$

for b)
since $\text{"}ln(0+)\text{"} = \lim_{x\to 0+}\ln x = -\infty$

$$\lim_{x\to 0^+}\frac{\ln x}{x^2+x-2} = \text{""}\frac{-\infty}{-2}\text{"}(NOL\prime H.) = +\infty$$

For 2)
using $a^b = e^{a\ln b}, a > 0$
by Chain and product Rules
$y = 3^{x^2} + x^{x^2} = e^{x^2\ln 3} + e^{x^2\ln x}$ so $y' = e^{x^2\ln 3}(x^2\ln 3)' + e^{x^2\ln x}(x^2\ln x)' =$
$$=e^{x^2\ln 3}(2x\ln 3) + e^{x^2\ln x}\left(2x\ln x + x^2\cdot\frac{1}{x}\right) = \ln 3\cdot e^{x^2\ln 3}(2x) + e^{x^2\ln x}(2x\ln x + x)$$
also
$$y' = 3^{x^2}(\ln 3)(x^2)' + u'$$
using log.diff. but only for $u = x^{x^2}$,and $(3^v)' = 3^v\ln 3\cdot v'$ with $v = x^2$
$u = x^{x^2}$ $\quad \ln u = \ln x^{x^2} = x^2\ln x \quad \dfrac{u'}{u} = 2x\ln x + x^2\cdot\dfrac{1}{x}$
$u' = x^{x^2}[2x\ln x + x]$ and $\quad y' = 3^{x^2}(\ln 3)2x + u'$...as above

For 3)
the correct formula for the amount present $A(t) = A_0e^{kt}$ where $k < 0$
t in years, $A_0 = 100\%$ half-life$T$ means that $50 = 100e^{kT}$

solve for k $\quad \frac{50}{100} = e^{kT}$ apply ln to both sides

$\ln \frac{1}{2} = Tk = 5k \qquad k = \frac{\ln 0.5}{5} = -0.1386294$

and

in % $\quad A(t) = 100e^{kt} \quad$ for k calculated above

now

$t = 2 \qquad A(2) = 100e^{k2} = 100e^{\frac{2 \ln 0.5}{5}} = 75.78582$ %