MATH 249
Worksheet #1 .

For 1 a)

$$|2x+1| \leq |x-2|$$

Since $|...|$ is always positive or zero we can square both sides and the sign of the inequality stays the same: $(2x+1)^2 \leq (x-2)^2$ since $|...|^2 = (...)^2$

Now

$4x^2+4x+1 \leq x^2-4x+4$ everything on one side: $3x^2+8x-3 \leq 0$

$(3x-1)(x+3) \leq 0$,thus split points (roots)are: $x=-3, \frac{1}{3}$,

testing: $--^{pos}---_{-3}--^{neg}--_{\frac{1}{3}}--^{pos}---$

so the solutions set is the closed interval $\left[-3, \frac{1}{3}\right]$

b)

$$\frac{3}{x+1} > \frac{1}{3} \quad \text{for } x \neq -1$$

everything on one side and common denominator: $\dfrac{3 \cdot 3-(x+1)}{(x+1)3} > 0$

simplify:

$\dfrac{9-x-1}{3(x+1)} > 0$ then $\dfrac{8-x}{(x+1)(3)} > 0.$ So split points are : $x = 8, -1$

testing: $--^{neg}---_{-1}--^{pos}--_{8}--^{neg}---$

solution set is the open interval $]-1, 8[$ or $(-1, 8)$.

For 2)

Complete the squares

$$x^2+4x+y^2-2y = 11 \qquad x^2+4x+4+y^2-2y+1 = 11+4+1$$

$(x+2)^2+(y-1)^2 = 16$ so $(x+2)^2+(y-1)^2 = 4^2$

thus

$r = 4$ and the point $C(-2, 1)$ is the centre of the circle.

For 3a) $|x+1|+2 > 0$

Since $|...|$ is always positive or zero $|x+1|+2$ is always positive for any x, so solution set is $(-\infty, +\infty)$.

b) $\dfrac{3}{x+1} \geq \dfrac{2}{x+3}$ for $x \neq -1, -3$

everything on one side and common denominator: $\dfrac{3(x+3)-2(x+1)}{(x+1)(x+3)} \geq 0$

simplify the top:

$\dfrac{3x+9-2x-2}{(x+1)(x+3)} \geq 0$ then $\dfrac{(x+7)}{(x+1)(x+3)} \geq 0.$

So split points are : $x = -7, -3, -1$

testing: $-^{neg}-_{-7}---^{pos}---_{-3}--^{neg}--_{-1}--^{pos}---$

solution set: $[-7, -3) \cup (-1, +\infty)$.

.

For 4)

For $l_1 : 3x+2y = 1$ $l_2 : 2y-3x = 0$ $l_3 : 3x-2y = 0$ and $l_4 : 2x-3y = 2$

find the slopes: $m_1 = -\frac{3}{2}$, $m_2 = \frac{3}{2}, m_3 = \frac{3}{2}, m_4 = \frac{2}{3}$

so $l_2 \| \, l_3$ since they have the same slope
and $l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

For 5 a) $\dfrac{1}{x+1} \leq 1 + x$ for $x \neq -1$

everything on one side and common denominator: $\dfrac{1-(x+1)^2}{(x+1)} \leq 0$,simplify:
$\dfrac{1-x^2-2x-1}{(x+1)} \leq 0$ then $\dfrac{-x\,(x+2)}{(x+1)} \leq 0$.So split points are : $x = 0, -2, -1$
testing $-^{pos} - -_{-2} - -^{neg} -_{-1} - -^{pos} - - -_{0} - -^{neg} - -$
solution set: $[-2,-1[\cup [0,+\infty[$ or $[-2,-1) \cup [0,+\infty)$.
b)

$|3x-2| > 0$

Since $|...|$ is always positive or zero we have to elliminate zero :$3x - 2 = 0$ for $x = \frac{2}{3}$
The solutions : $x \neq \frac{2}{3}$ or $\left]-\infty, \frac{2}{3}\right[\cup \left]\frac{2}{3}, +\infty\right[$ or $\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, +\infty\right)$
For 6)
$\perp$ to x-axis means a vertical line so $x = -1$ (y is any).
line parallel to the x-axis means a horizontal line so $y = 3$ (x any)
For 7 a)

$3x + 7 > x^2$

Everything on one side: $0 > x^2 - 3x - 7$ now find the roots ,
first discriminant $D = (-3)^2 - 4 \cdot 1 \cdot (-7) = 9 + 28 = 37$,
so using the quadratic formula roots are
$x_1 = \frac{3-\sqrt{37}}{2} \doteq -1.54$ and $x_2 = \frac{3+\sqrt{37}}{2} \doteq 4.54$
Now testing : $- -^{pos} - -_{x_1} - -^{neg} - - -_{x_2} - -^{pos} -$
OR parabola open up and it is below the x-axis if $x \in \,]-1.54, 4.54[$.
(between the roots $x \in (x_1, x_2)$)
b)

$\dfrac{x}{2} < \dfrac{2}{x+3}$. for $x \neq -3$

everything on one side and common denominator: $\dfrac{x(x+3) - 2 \cdot 2}{2(x+3)} < 0$
simplify:
$\dfrac{x^2+3x-4}{2(x+3)} < 0$ then $\dfrac{(x+4)\,(x-1)}{2(x+3)} < 0$. So split points are : $x = -4, -3, 1$
testing:$-^{neg} - -_{-4} - -^{pos} - -_{-3} -^{neg} - -_{1} -^{pos} - -$
solution set: $]-\infty,-4[\cup]-3, 1[$ or $(-\infty,-4) \cup (-3,-1)$.
For 8)

$x^2 - 6x + y^2 = 7 \qquad x^2 + y^2 + 2y = 15$

Complete the squares :
$x^2 - 6x + 9 + y^2 = 7 + 9$, $x^2 + y^2 + 2y + 1 = 1 + 15$
So the equations are:
$(x-3)^2 + y^2 = 16 \qquad x^2 + (y+1)^2 = 16$
thus radii are the same $r = 4$, the centres are points $(3, 0)$ and $(0, -1)$.
.
For 9) for $h \neq 0, 7$

find the common denominator first, then $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{cb}$

$$\frac{\frac{3h+4}{7-h} - \frac{4}{7}}{\frac{25h}{7}} = \frac{\frac{7(3h+4)-4(7-h)}{7(7-h)}}{\frac{25h}{7}} = \frac{21h + 28 - 28 + 4h}{7\,(7-h)} \cdot \frac{7}{25h} = \frac{1}{7-h}.$$

For 10) for $x \neq -1$ as in 9)

$$\frac{1}{1 + \frac{1}{x+1}} = \frac{1}{\frac{x+1+1}{x+1}} = \frac{x+1}{x+2}$$ and for $x \neq -2$.

For 11)

factor out the polynomials

$$\frac{x^3 + 5x^2 + 6x}{12 + x - x^2} = \frac{x(x^2 + 5x + 6)}{-(x^2 - x - 12)} = \frac{x(x+3)\,(x+2)}{-(x-4)\,(x+3)} = -\frac{x\,(x+2)}{(x-4)}$$

for $x \neq -3, 4$.

For 12)

factor out the polynomials ,then common denom.

$$\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} = \frac{x}{(x+2)(x-1)} - \frac{2}{(x-4)(x-1)} = \frac{x\,(x-4) - 2\,(x+2)}{(x+2)(x-1)\,(x-4)} =$$

$$= \frac{x^2 - 6x - 4}{(x+2)(x-1)\,(x-4)}$$ for $x \neq -2, 1, 4$.

For 13)

$$\frac{x}{x-1} < \frac{1}{x+1} \qquad \text{for } x \neq \pm 1$$

everything on one side and common denominator: $\frac{x\,(x+1) - (x-1)}{(x-1)\,(x+1)} < 0$

simplify $\frac{x^2+1}{(x-1)\,(x+1)} < 0$ the top has NO real roots,always positive

thus only two split points $x = \pm 1$

Now testing : $--^{pos}--_{-1}--^{neg}---_{1}--^{pos}-$

the solution set is $(-1, 1)$.

For 14)

$$\frac{x}{x-1} > \frac{4}{x} \qquad \text{for } x \neq 1, 0$$

everything on one side and common denominator: $\frac{x^2 - 4\,(x-1)}{(x-1)\,(x)} > 0$

simplify $\frac{x^2 - 4x + 4}{x\,(x-1)} > 0$ $\frac{(x-2)^2}{x\,(x-1)} > 0$

the top has a double root, always positive or zero

the split points $x = 0, 1, 2$ but only $x = 0, 1$ are switch points

Now testing :

$$--^{pos}--_{0}--^{neg}---_{1}--^{pos}-_{2}-^{pos}---$$

check the split points!!

the solution set is $(-\infty, 0) \cup (1, 2) \cup (2, +\infty)$.