

- How much money one has to invest today at the interest of 3% compounded quaterly to get \$10,000 in 10 years?
- Find the domain and derivative of $f(x) = (3x)^\pi + \pi^{3x} + (\pi x)^{3x}$.
- Find y' if $y = 2^{x^4} + \frac{2}{x^4} + \left(\frac{1}{x}\right)^x$, for $x > 0$.
- How long does it take to double your investment if the interest of 7 % is compounded
 - yearly?
 - monthly?
- In the first few weeks after birth, a baby gains weight at a rate proportional to its weight. A baby weighing 4kg at birth weighs 4.4kg after 2 weeks. How much did it weigh 4 days after birth ?
- For $f(x) = 3^x \ln \frac{3}{x}$ find $f'(3)$.
- Find y' if $y = x^{x^2} + \ln \frac{1}{1-x}$, for $0 < x < 1$.
- After 3 days a sample of radon-222 decayed to 58% of its original amount. What is half-life of radon-222?
- Evaluate
 - $\lim_{x \rightarrow +\infty} \frac{x}{2^x - 1}$
 - $\lim_{x \rightarrow -\infty} \frac{x}{2^x - 1}$
 - $\lim_{x \rightarrow 0} \frac{x}{2^x - 1}$
- Evaluate
 - $\lim_{x \rightarrow \infty} \frac{x^2}{e^{3x}}$
 - $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{3x}}$
 - $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}$
 - $\lim_{x \rightarrow 0^+} \frac{(\ln x)^2}{x}$.
- Solve for x : $\frac{1}{3^{x+1}} = \frac{5}{4^x}$.
- Solve for x:
 - $\frac{1}{2} \ln(x + 3) + 1 = 0$;
 - $3^{x^2} = 9^{x-3}$.

SOLUTION For1)

the formula to use is $A(t) = A_0 \left(1 + \frac{p}{100n}\right)^{nt}$ where $t = 10, n = 4, p = 3$

$A = 10,000$ $A_0 = ?$ is the initial amount to invest so

$$10000 = A_0 \left(1 + \frac{3}{400}\right)^{40} = A_0 \left(\frac{403}{400}\right)^{40} \text{ multiply by reciprocal to isolate } A_0$$

$$A_0 = 10000 \left(\frac{400}{403}\right)^{40} = \$7416.48$$

For 2)

$$\text{for } x > 0 \quad f(x) = (3x)^\pi + \pi^{3x} + (\pi x)^{3x} = 3^\pi x^\pi + (\pi^3)^x + e^{3x \ln \pi x}$$

power + exp.f + e^u so by Chain Rule $(e^u)' = e^u u'$

$$f'(x) = 3^\pi (x^\pi)' + [(\pi^3)^x]' + e^{3x \ln \pi x} [3x \ln(\pi x)]' \text{ (Pr.R.)}$$

$$= \pi 3^\pi x^{\pi-1} + (\pi^3)^x \ln \pi^3 + e^{3x \ln \pi x} \left[3 \ln \pi x + 3x \cdot \frac{1}{\pi x} \cdot \pi \right] =$$

$$= \pi 3^\pi x^{\pi-1} + 3 (\pi^3)^x \ln \pi + e^{3x \ln \pi x} [3 \ln \pi x + 3]$$

ALSO

change all terms into e^u and then Chain rule

$$f(x) = (3x)^\pi + \pi^{3x} + (\pi x)^{3x} = e^{\pi \ln 3x} + e^{3x \ln \pi} + e^{3x \ln \pi x}$$

so

$$f'(x) = e^{\pi \ln 3x} (\pi \ln 3x)' + e^{3x \ln \pi} (3x \ln \pi)' + e^{3x \ln \pi x} (3x \ln \pi x)' =$$

$$= e^{\pi \ln 3x} \left(\pi \cdot \frac{3}{3x} \right) + e^{3x \ln \pi} (3 \ln \pi) + e^{3x \ln \pi x} \left[3 \ln \pi x + 3x \cdot \frac{1}{\pi x} \cdot \pi \right] =$$

$$= \frac{\pi}{x} (3x)^\pi + (3 \ln \pi) \pi^{3x} + [3 \ln \pi x + 3] (\pi x)^{3x}$$

For 3)

$$y' = (e^{x^4 \ln 2})' + (2x^{-4})' + \left(e^{x \ln \frac{1}{x}}\right)' = e^{x^4 \ln 2} (x^4 \ln 2)' - 8x^{-5} + e^{-x \ln x} \cdot (-x \ln x)' =$$

$$= 4x^3 \ln 2 e^{x^4 \ln 2} - 8x^{-5} - e^{-x \ln x} \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = 4x^3 \ln 2 e^{x^4 \ln 2} - 8x^{-5} - e^{-x \ln x} (\ln x + 1)$$

OR

by log.diff.BUT only for the last term $u = \left(\frac{1}{x}\right)^x = e^{x \ln \frac{1}{x}}$ so

$$\ln u = x \ln \frac{1}{x} = x \ln x^{-1} = -x \ln x$$

and

$$\frac{1}{u} \cdot u' = -\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = -(\ln x + 1) \text{ so } u' = -\left(\frac{1}{x}\right)^x (\ln x + 1)$$

for the first term we can use $(2^u)' = 2^u \cdot \ln 2 \cdot u'$ so $(2^{x^4})' = 2^{x^4} \cdot \ln 2 \cdot 4x^3$

and the second term is just a power so $(2x^{-4})' = -8x^{-5}$.

Together

$$y' = 4x^3 \ln 2 \cdot 2^{x^4} - 8x^{-5} - \left(\frac{1}{x}\right)^x (\ln x + 1) \quad \text{as above.}$$

For 4a)

we have to solve for t $2A_0 = A_0 \left(1 + \frac{7}{100}\right)^t$ cancel A_0

thus $2 = (1.07)^t$ take \ln of both sides:

$$\ln 2 = t \ln 1.07, \text{ so } t = \frac{\ln 2}{\ln 1.07} = 10.24$$

We need 10 years and almost 3 months.

For 4b)

we have to solve for t $2A_0 = A_0 \left(1 + \frac{7}{1200}\right)^{12t}$

as above

$$\text{thus } 2 = \left(\frac{1207}{1200}\right)^{12t} \text{ and}$$

$$\ln 2 = 12t \ln \frac{1207}{1200} \quad t = \frac{\ln 2}{12(0.0058163)} = 9.929.$$

So this time we need less than 10 years.

For 5)

for the weight after t **weeks** $W(t) = 4e^{kt}$, given $W(2) = 4.4 = 4e^{2k}$
solve for k :

$$\frac{4.4}{4} = e^{2k}, \text{ apply ln to both sides: } \ln 1.1 = 2k, \text{ so } k = \frac{\ln 1.1}{2}$$

Now, after 4 days = $\frac{4}{7}$ of a week ,

so

$$W = 4e^{k \cdot \frac{4}{7}} = 4e^{\frac{2}{7} \cdot \ln 1.1} = 4.11 \text{ kg.}$$

OR

$W(t) = 4e^{kt}$ where we measure time t **in days**

then 2 weeks is 14 days

$$W(14) = 4.4 = 4e^{14k} \text{ then } k = \frac{\ln 1.1}{14}$$

and for $t = 4$ $W = 4e^{k \cdot 4} = 4e^{4 \cdot \frac{\ln 1.1}{14}} = (\text{as above}) = 4.11 \text{ kg}$

For 6)

first simplify $f(x) = 3^x \cdot (\ln 3 - \ln x)$, then use Product Rule

$$f'(x) = (3^x)' (\ln 3 - \ln x) + 3^x (\ln 3 - \ln x)' = 3^x \ln 3 (\ln 3 - \ln x) + 3^x \left(\frac{-1}{x}\right)$$

since $(\ln 3)' = 0$ then substitute $x = 3$

$$\text{and } f'(3) = 3^3 \ln 3 (\ln 3 - \ln 3) + 3^3 \left(\frac{-1}{3}\right) = -9.$$

For 7)

simplify first

$$y = x^{x^2} + \ln \frac{1}{1-x} = e^{x^2 \ln x} - \ln(1-x) \quad y' = e^{x^2 \ln x} (x^2 \ln x)' - \frac{1}{1-x} (1-x)' = \\ = e^{x^2 \ln x} (2x \ln x + x) + \frac{1}{1-x}$$

OR you can use log.diff. but only for the first part

$$u = x^{x^2} \quad \ln u = x^2 \ln x \quad \frac{u'}{u} = 2x \ln x + x^2 \cdot \frac{1}{x} \quad u' = x^{x^2} (2x \ln x + x)$$

$$\text{and } y' = u' + \frac{1}{1-x}.$$

For 8)

the correct formula $A(t) = A_0 e^{kt}$ where $k < 0$, t in **days**, $A_0 = 100\%$

first info if $t = 3$ $58 = 100e^{3k}$ solve for k $\ln \frac{58}{100} = 3k$

$$k = \frac{\ln 0.58}{3} = -0.1815757$$

so $A(t) = 100e^{kt}$ for k calculated above ;

now half-life T means we got 50%

$$50 = 100e^{kT} \quad \text{where } k = \frac{\ln 0.58}{3} \quad \frac{50}{100} = e^{kT}$$

$$\text{solve for } T \quad \ln 0.5 = kT \quad T = \frac{3 \ln 0.5}{\ln 0.58} = 3.8174 \text{ days}$$

For 9) for a)

$\lim_{x \rightarrow +\infty} \frac{x}{2^x - 1}$ the type is " $\frac{+\infty}{+\infty}$ " so we can use L'Hop.rule

$$\lim_{x \rightarrow +\infty} \frac{(x)'}{(2^x - 1)'} = \lim_{x \rightarrow +\infty} \frac{1}{2^x \ln 2} = \frac{1}{\infty} = 0$$

for b)

$\lim_{x \rightarrow -\infty} \frac{x}{2^x - 1}$ since " $2^{-\infty}$ " = 0 No L'H.R. $\lim_{x \rightarrow -\infty} \frac{x}{2^x - 1} = \frac{-\infty}{-1} = +\infty$.

for c)

$\lim_{x \rightarrow 0} \frac{x}{2^x - 1}$ the type is " $\frac{0}{0}$ " so L'Hop.Rule again $\lim_{x \rightarrow 0} \frac{x}{2^x - 1} = \lim_{x \rightarrow 0} \frac{1}{2^x \ln 2} = \frac{1}{\ln 2}$

For 10)

for a)

the type is " $\frac{\infty}{\infty}$ " so use L'Hop.Rule

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^{3x}} = \lim_{x \rightarrow \infty} \frac{2x}{3e^{3x}} \text{ (again)} = \lim_{x \rightarrow \infty} \frac{2}{9e^{3x}} = \frac{2}{\infty} = 0$$

for b)

the type is " $\frac{\infty}{0^+}$ " since " $e^{-\infty}$ " = 0

so $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{3x}} = +\infty(+\infty) = +\infty$ since " $\frac{1}{0^+}$ " = $+\infty$. No L'H.R.

also $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{3x}} = \lim_{x \rightarrow -\infty} x^2 e^{-3x} = +\infty \cdot e^{+\infty} = +\infty$

for c) $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} = \frac{\infty}{\infty}$ (L'H.R.) = $\lim_{x \rightarrow \infty} \frac{2(\ln x) \frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} =$

again L'H.R. = $\lim_{x \rightarrow \infty} \frac{\frac{2}{x}}{1} = \frac{2}{\infty} = 0$

for d) $\lim_{x \rightarrow 0^+} \frac{(\ln x)^2}{x} = \frac{\infty}{0^+} = \lim_{x \rightarrow 0^+} (\ln x)^2 \cdot \frac{1}{x} = (-\infty)^2 (+\infty) = +\infty$
(No L'H.R.)

For 11)

cross multiply first, so $4^x = 5 \cdot 3^{x+1}$, then apply ln to both sides

$$\ln 4^x = \ln (5 \cdot 3^{x+1}) = \ln 5 + \ln 3^{x+1}$$

thus $x \ln 4 = \ln 5 + (x+1) \ln 3$ $x \ln 4 = \ln 5 + x \ln 3 + \ln 3$

and $x \ln 4 - x \ln 3 = \ln 5 + \ln 3$

So $x(\ln 4 - \ln 3) = \ln (5 \cdot 2)$ and finally $x \ln \frac{4}{3} = \ln 10$

$$x = \frac{\ln 10}{\ln \frac{4}{3}} = \frac{\ln 10}{\ln 4 - \ln 3}$$

For 12a)

$$\frac{1}{2} \ln(x+3) = -1 \quad \ln(x+3) = -2$$

then exp.f. to both sides and $e^{\ln(x+3)} = e^{-2}$

$$(x+3) = e^{-2}$$

and so $x = e^{-2} - 3$

OR $\frac{1}{2} \ln(x+3) = \ln \sqrt{x+3} = -1$

then apply exp.f to get $\sqrt{x+3} = e^{-1}$

then square to get the above.

b)

Take log of both sides: $\ln 3^{x^2} = \ln 9^{x-3}$

$$x^2 \ln 3 = (x-3) \ln 9 = (x-3) \ln 3^2 = (x-3) \cdot 2 \ln 3$$

cancel $\ln 3$ and $x^2 = 2(x-3) = 2x-6$

everything on one side: $x^2 - 2x + 6 = 0$, discriminant of this polynomial is

$D = (-2)^2 - 4 \cdot 1 \cdot 6 = -20$ so no real roots exist and the problem has **NO solution**.

Also we can change both sides to the same base: $3^{x^2} = (3^2)^{x-3} = 3^{2x-6}$

and by comparing the exponents we get the same quadratic equation.