THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL Handout
MATH 249-01

1. Evaluate the limits:

(a) $\lim\limits_{x\to\pi} \dfrac{\cos\left(\frac{x}{2}\right)}{\pi - x}$ $\quad(b)$ $\lim\limits_{x\to-\infty} \dfrac{\cos\left(\frac{x}{2}\right)}{\pi - x}$ $\quad(c)$ $\lim\limits_{x\to 0^+} x\ln x$

2. Find the domain and the derivative of f of

   (a) $f(x) = \frac{x}{3}e^{-\sin\left(\frac{3}{x}\right)}$

   (b) $f(x) = \dfrac{\ln(2x-3)}{e^{-x^2}}$

3. **A**

   Sketch the graph of $y = \dfrac{1}{x^2+x-2}$ if $y' = -\dfrac{2x+1}{(x^2+x-2)^2}$and $y'' = \dfrac{6x^2+6x+6}{(x^2+x-2)^3}$ i.e.

   (a) ?nd the domain, range, vertical and horizontal asymptotes, x and y intercepts;
   (b) ?nd the intervals where f is increasing or decreasing;
   (c) ?nd the intervals where f is concave down or up

   3B

   Sketch the graph of $y = x(4-x)^3$.Indicate where the function is increasing,decreasing, concave up,concave down;?nd the domain and range.

4. (a) Find the tangent approximation (linearization) of

$$f(x) = \frac{1}{\sqrt{2x^2+1}} \text{ around } x_0 = 2.$$

   (b) Use it to estimate $\frac{1}{\sqrt{3}}$.

5. **A**

   Sketch a graph of <u>one</u> function f satisfying all the following conditions:

   (a) f is de?ned for all x, continuous there except
   (b) f is discontinuous at $x = 2, 4$ where $\lim\limits_{x\to 4^-} f(x) = f(4) = 0$, $\quad x = 2$ is a vertical asymptote.
   (c) $y = 3$ is a horizontal asymptote and $\lim\limits_{x\to-\infty} f(x)$ <u>does not exist</u>,

(d) f is increasing on $(3,4)$ and on $(4,+\infty)$, f is decreasing on $(0,2)$ and on $(2,3)$, and $f'(x)=0$ for all $x\in(-2,0)$;

(e) f is concave up on $(0,1)$ and on $(3,4)$; f is concave down on $(1,2)$, on $(2,3)$ and on $(4,+\infty)$

(f) absolute maximum value is 6, and local minimum value is -2.

B

Sketch a graph of <u>one</u> function f satisfying all the following conditions:

(a) f is de?ned on $(0,+\infty)$, continuous there except

(b) f is discontinuous at $x=1,2,3$ where $\lim_{x\to2}f(x)=3$, $\lim_{x\to3}f(x)$ DNE (does not exist)

(c) $x=1$ is a vertical asymptote; $y=2$ is a horizontal asymptote;

(d) f is increasing on $(2,3)\,,(3,4)$and on $(5,+\infty)$; f is decreasing on $(0,1)$and on $(4,5)$

and $f'(x)=0$ for all $x\in(1,2)$;

(e) f is concave up on $(2,3)$ and on $(4,6)$;

f is concave down on $(6,+\infty)$ and on$(3,4)$;

(f) absolute maximum value is 5, and local minimum value is -1.

C

Sketch a graph of <u>one</u> function f satisfying all the following conditions:

(a) f is de?ned on $[-1,+\infty)$ and continuous there except ;

(b) f is discontinuous at $x=1,3$ where $\lim_{x\to1^-}f(x)$ DNE;

(c) $x=3$ is a vertical asymptote; $y=2$ is a horizontal asymptote;

(d) f is increasing on $(-1,0)$ and on $(3,+\infty)$, f is decreasing on $(0,1)$ and on $(2,3)$, and $f'(x)=0$ for all $x\in(1,2)$;

(e) f is concave up on $(-1,0)\,,(3,4)$ and on $(0,1)$; f is concave down on $(2,3)$ and on $(4,+\infty)$

(f) absolute maximum value is 7,amd local minimum value is 0.;

6. **A**

A box with a square base(bottom) and a top(lid) has a volume of 18 m^3.Find the dimensions of the most economical box if the material

for the base and lid costs \$2 per m^2 and the material for the sides \$3 per m^2.

B

A landscape architect plans to enclose a 280 m^2 rectangular region in a botanical garden.

She will use shrubs costing \$25.00 per meter along three sides and fencing costing \$10.00 per meter along the fourth side.

Find the dimensions of the region to minimize the total cost.

7. Find (a) for $x > 0$ $\displaystyle\int \frac{3\sqrt{x}-5}{x\sqrt{x}}dx =$ (b) $\displaystyle\int 2x^3\sqrt{2x^2+3}dx.$

8. Evaluate (a) $\displaystyle\int_{\frac{1}{2}}^{1} \frac{3^{\frac{1}{x}}}{x^2}dx$ (b) $\displaystyle\int_0^1 \frac{4x+3}{3-2x}dx.$

(a) Solve for x : $2\ln(x+1) - \ln(4x) = 0.$

(b) Solve for x : $\displaystyle\frac{1}{2^{2x+1}} = \frac{5}{4^{3x}}.$