University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Calculus 1 Fall 2009

Worksheet 4(Continuity)

1. Determine whether or not the function given in each case is continuous at the given point. Give reasons for your answer.

a. $f(x) = \begin{cases} x^3 + x^2 & x \le -2 \\ 2x^2 - 4 & x > -2 \end{cases}$ at x = -2.

b. $f(x) = \begin{cases} |x^2 - 4| & -2 \le x \le 2 \\ 2x - 4 & x > 2 \\ 3x + 4 & x < -2 \end{cases}$ at x = 2 and at x = -2.

c. $f(x) = \begin{cases} \dfrac{x^3 - 9x}{x^2 + x - 12} & x > 3 \\ \dfrac{10}{7} & x = 3 \\ \dfrac{2x^2}{7} & x < 3 \end{cases}$ at x = 3.

d. $f(x) = \begin{cases} x + \dfrac{1}{x} & x < 0 \\ -x^3 & x \ge 0 \end{cases}$ at x = 0.

e. $f(x) = \begin{cases} x + \dfrac{1}{x} & x < 0 \\ -2 & x = 0 \\ -\dfrac{1}{x^3} & x > 0 \end{cases}$ at x = 0.

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2. In each case determine values of a so that the function given is continuous.

a. $$f(x) = \begin{cases} 3x^3 - 4x^2 + a & x \le -2 \\ 4x^2 - 1 & x > -2 \end{cases}$$

b. $$f(x) = \begin{cases} \dfrac{x^3 + x^2 - ax}{x^2 - 1} & x \le -2 \\ 2x^2 + 3x - 4 & x > -2 \end{cases}$$

3. Determine values of a and b so that the function given is continuous.

$$f(x) = \begin{cases} 3x^3 - bx^2 + a & x < -2 \\ 1 & x = -2 \\ ax^2 + bx - 1 & x > -2 \end{cases}$$

4. Show that the cubic equation
$$x^3 + x^2 - x - 4 = 0$$
has a root in the interval (1,2).

5. If $f(x) = x^3 + x - 1$,
show that f has a zero between $x = 0$ and $x = 1$

6. Show that $f(x) = x^3 - 15x + 1$
has at least three zeros in the closed interval $[-4, 4]$.