

Worksheet 5a (Answers)  
[Derivatives and applications]

1. a. Determine the points on the curve  
 $y = 2x^3 - 9x^2 - 24x + 4$   
where the tangent line to the curve is parallel to the x-axis.  
*The points are (-1,17) and (4,-108).*
- b. Determine the points on the curve  
 $y = 3x^4 - 2x^3 - 9x^2 + 1$   
where the tangent line to the curve is parallel to the x-axis.  
*The points are (0,1), (-1,-2), and (3/2,-173/16)*
- c. Find the equation of the tangent and normal lines to the curve  
 $y = x + \frac{1}{\sqrt{x}}$   
at the point where  $x = 4$ .  
*The tangent line is  $15x - 16y = -12$ ;  
The normal line is  $32x + 30y = 263$*
- d. Find the equation of the tangent and normal lines to the curve  
 $y = x^4 + 2x^3 + x^2 + x + 1$   
at the point where the tangent line has slope equal to  
1  
*The tangent line has slope 1 at the points (0,1), (-1/2,9/16), and (-1,0).  
at (0,1)  
The tangent line is  $y = x + 1$   
The normal line is  $y = -x + 1$   
at (-1/2,9/16),  
The tangent line is  $y = x + \frac{17}{16}$   
The normal line is  $y = -x + \frac{1}{16}$   
at (-1,0)  
The tangent line is  $y = x + 1$   
The normal line is  $y = -x - 1$*
- e. Find the equation of the tangent and normal lines to the curve  
 $y = 3x^4 + 4x^3 + 12x^2 - 22x + 1$

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at the points on the curve where the tangent line is parallel to the straight line  $y = 2x - 5$ .

**This occurs at (1, -2).**

**The tangent line is  $y = 2x - 4$**

**The normal line is  $2y = -x - 3$**

2. Use the definition of the derivative as a limit to determine  $f'(x)$  given that

a.  $f(x) = \sqrt{x^2 + 4}$

b.  $f(x) = \frac{1}{x^2 - 4}$

c.  $f(x) = \frac{4x + 5}{3 - 5x}$

d.  $f(x) = \frac{1}{\sqrt{4 - x^2}}$

In each case check your answer by applying the rules for differentiation.

3. In each case, determine  $f'(x)$  ~~g'(x)~~.

i.  $f(x) = \sin(x^4 + x^3)$   
 $f'(x) = (4x^3 + 3x^2) \cos(x^4 + x^3)$

ii.  $f(x) = \tan(\sqrt{4 - x^3})$   
 $f'(x) = -\frac{3x^2}{2\sqrt{4 - x^3}} \sec^2(\sqrt{4 - x^3})$

iii.  $f(x) = \sec(x^{2/3} - x)$   
 $f'(x) = \left(\frac{2}{3}x^{-1/3} - 1\right) \sec\left(x^{2/3} - x\right) \tan\left(x^{2/3} - x\right)$

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iv.  $f(x) = \operatorname{cosec}(\sqrt{x^4 + 1})$   
 $f'(x) = -\frac{2x^3}{\sqrt{x^4 + 1}} \operatorname{cosec} \sqrt{x^4 + 1} \cot \sqrt{x^4 + 1}$

v.  $f(x) = \cos(\cos(\cos x))$   
 $f'(x) = [-\sin(\cos(\cos x))] [-\sin(\cos x)] [-\sin x]$   
 $= -\sin(\cos(\cos x)) \cdot \sin(\cos x) \cdot \sin x$

vi.  $f(x) = \cot(\sin(x^3 + 1))$   
 $f'(x) = [-\operatorname{cosec}^2(\sin(x^3 + 1))] \cdot \cos(x^3 + 1) \cdot (3x^2)$   
 $= -3x^2 \operatorname{cosec}^2(\sin(x^3 + 1)) \cos(x^3 + 1)$

vii.  $f(x) = \sqrt{x^2 + \sqrt{x^2 + \sqrt{x^2 + 1}}}$   
 $f'(x) = \left[ \frac{1}{2} \left( x^2 + \sqrt{x^2 + \sqrt{x^2 + 1}} \right)^{-\frac{1}{2}} \right] \cdot \left[ 2x + \frac{1}{2} \left( x^2 + \sqrt{x^2 + 1} \right)^{-\frac{1}{2}} \cdot \left( 2x + \frac{1}{2} \left( x^2 + 1 \right)^{-\frac{1}{2}} \cdot (2x) \right) \right]$

4. Determine all points on each graph of the function given where the tangent line is parallel to the x-axis.

a  $y = x^4 - x^2 + 1$

The points are  $(0,1)$ ,  $\left(\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$ ,  $\left(-\frac{1}{\sqrt{2}}, \frac{3}{4}\right)$

b  $y = \sin 2x - 2 \sin x$

The points are  $\left(2k\pi + \frac{2\pi}{3}, -\sqrt{3}\right)$ ,  $\left(2k\pi + \frac{4\pi}{3}, \sqrt{3}\right)$ ,  $(2k\pi, 0)$   
 for all integers  $k$ .

c  $y = \tan x + \cot x$

The points are  $\left(k\pi + \frac{\pi}{4}, 2\right)$ ,  $\left(k\pi + 3\frac{\pi}{4}, -2\right)$  for all integers  $k$ .

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5. Determine whether or not the given curve has a tangent line which is parallel to the  $x$ -axis:

a  $y = 2x^3 + 3x^2 + 6x + 12$

*Here,  $y' = 6(x^2 + x + 1)$ .*

*since  $x^2 + x + 1 = 0$  has no real solutions,*

*There is no tangent line parallel to the  $x$ -axis.*

b  $y = 2x^3 - x^2 + 2x - 1$

*Here,  $y' = 6x^2 - 2x + 2$ .*

*since  $6x^2 - 2x + 2 = 0$  has no real solutions,*

*There is no tangent line parallel to the  $x$ -axis.*