THE UNIVERSITY OF CALGARY
MATHEMATICS 249
FINAL EXAMINATION, FALL 2003
TIME: 2 HOURS

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11	
Total (max. 65)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [65]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\dfrac{d}{dx}\left(\dfrac{\cos^2 x - e^{3x}}{\sin x}\right)$.

[5] 2. Find $\dfrac{d}{dx}\left(x^{1/4}\ln(7-6x)\right)$.

[6] 3. USE THE DEFINITION OF DERIVATIVE to find $\frac{d}{dx}\left(\sqrt{7x}\right)$.

[6] 4. Use implicit differentiation to find dy/dx where $y^2 \tan(x + y^2) = 4x$.

[8] 5. An object moves along the number line so that its position at any time t is given by

$$p(t) = \frac{3t - 4}{t^2 + 1}\ .$$

(a) Show that the velocity of the object at time t is given by $v(t) = -\dfrac{3t^2 - 8t - 3}{(t^2 + 1)^2}$.

(b) Find all times t when the velocity is zero.

(c) Find the acceleration of the object at time $t = 0$.

[9] 6. For the function $f(x) = x(6 - x)^3$,
(a) show that $f'(x) = -2(6 - x)^2(2x - 3)$.

Then find (b) the critical numbers; (c) the intervals of increase and decrease; (d) all local maxima and local minima.

[5] 7. For the function $f(x) = \dfrac{x}{e^{2x}}$, you are given that

$$f'(x) = \frac{1-2x}{e^{2x}} \qquad \text{and} \qquad f''(x) = \frac{4x-4}{e^{2x}} \ .$$

Find the intervals on which $f(x)$ is concave up and where it is concave down. Then find all points of inflection.

[5] 8. Find all constants k so that the function

$$f(x) = \begin{cases} \cos 3x & \text{if } x < 0, \\ (3\cos x) + k & \text{if } x \geq 0 \end{cases}$$

is continuous at $x = 0$. Also, for each such value of k, determine whether $f(x)$ is differentiable at $x = 0$.

[5] 9. Find and simplify $\int \frac{(\ln x)^5}{x}\, dx$.

[5] 10. Find and simplify $\int_0^{\pi/2} (\sin x + 249 \cos x)\, dx$.

[6] 11. Find the point on the curve $y = \dfrac{4}{\sqrt{x}}$ which is closest to the origin $(0, 0)$.

[65]