

UNIVERSITY OF CALGARY  
DEPARTMENT OF MATHEMATICS AND STATISTICS  
MATHEMATICS 249 — L05  
ANSWERS TO MIDTERM EXAM(b) NOV. 10, 2009

Total Marks = 80.

Duration = 75 minutes.

Work all problems. Marks are shown in brackets.  
NO CALCULATORS OR FORMULA SHEETS.

- [5] 1. Draw the graph of  $\ln x$ .  
Locate all  $x$  and  $y$  intercepts on the graph and give their coordinates. Do the same for the point at  $x = 1/e^2$ .  
The graph was supposed to be drawn from memory.  
See your lecture notes.

- [3] 2. State the  $\varepsilon - \delta$  definition of  $\lim_{x \rightarrow a} f(x) = L$ .  
This question was supposed to be answered from memory.  
See your lecture notes.

- [9] 3. Find the slope of the tangent line to the graph of

$$y \cos(x + y) = x^2 + \cos y$$

at the point  $(0, \pi/2)$ .

$$\begin{aligned} y' \cos(x + y) - y \sin(x + y)(1 + y') &= 2x - y' \sin y \\ -\frac{\pi}{2}(1 + y') &= -y' \end{aligned}$$

$$y' = \frac{\pi/2}{1 - \pi/2}$$

- [10] 4. Let  $f(x) = x^2 + x \sin x$ . Find the linear approximation to  $f$  at the point  $x_0 = \pi/2$ .

$$f(x_0) = f(\pi/2) = \left(\frac{\pi}{2}\right)^2 + \frac{\pi}{2}$$

$$f'(x) = 2x + \sin x + x \cos x \quad f'(x_0) = f'(\pi/2) = \pi + 1$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) = \left(\frac{\pi}{2}\right)^2 + \frac{\pi}{2} + (\pi + 1)(x - \pi/2)$$

5. For  $x < 1$  the graph of a certain function  $f$  is the curve  $y = 10x^{1/5}$ . For  $x \geq 1$  the graph of  $f$  is the line joining the points  $(1, 10)$  and  $(6, y_0)$ .

[3] (a) Find  $\frac{f(1+h) - f(1)}{h}$  when  $h < 0$ .

$$\frac{f(1+h) - f(1)}{h} = \frac{10(1+h)^{\frac{1}{5}} - 10}{h}$$

[3] (b) Find  $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$ .

$$\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h} = y'(1) = 2$$

[3] (c) Find a value of  $y_0$  such that  $f'(1)$  exists.

$$\frac{y_0 - 10}{6 - 1} = 2 \quad y_0 = 20$$

[9] 6. Find the limit

$$\lim_{h \rightarrow 0} \frac{\ln \left[ (1+h)^2 + 2 \right] - \ln 3}{h}$$

$$\lim_{h \rightarrow 0} \frac{\ln \left[ (1+h)^2 + 2 \right] - \ln 3}{h} = f'(1) = \frac{2}{3} \quad \text{where } f(x) = \ln(x^2 + 2) \quad f'(x) = \frac{2x}{x^2 + 2}$$

or use L'Hopital's rule

$$\lim_{h \rightarrow 0} \frac{\frac{d}{dh} [\ln \left[ (1+h)^2 + 2 \right] - \ln 3]}{\frac{d}{dh} h} = \lim_{h \rightarrow 0} \frac{\frac{2(1+h)}{(1+h)^2 + 2} - 0}{1} = \frac{2}{3}$$

**Each of the following multiple choice questions has one and only one correct answer. Write the letter corresponding to your answer in the box provided. Each multiple choice question is worth 7 marks.**

7.  $\lim_{x \rightarrow -\infty} \frac{1 - x + 5x^7 + x^9}{3 + x - 4x^9 - 8x^6}$  is (b)

(a)  $-\frac{1}{3}$

(b)  $-\frac{1}{4}$

(c)  $\frac{1}{4}$

(d)  $-\frac{1}{8}$

(e)  $\frac{1}{8}$



8. The definition of the statement a function  $f$  is continuous at the point  $a$  is (e)



- (a)  $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$
- (b)  $f'(a)$  exists
- (c)  $f'(a)$  exists and  $\lim_{x \rightarrow a} f(x) = f(a)$ .
- (d)  $f'(a)$  exists and  $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$ .
- (e)  $\lim_{x \rightarrow a} f(x) = f(a)$

9. For  $f(x) = x^2 e^{(x^3+1)}$ ,  $f'(x) =$  (d)



- (a)  $2xe^{3x^2}$
- (b)  $6x^3 e^{(x^3+1)}$
- (c)  $2xe^{(x^2+1)} + x^3 e^{(x^3+1)}$
- (d)  $x(2 + 3x^3) e^{(x^3+1)}$
- (e) none of the above

10. For every differentiable function  $f$ , its derivative at  $x = a$ , i.e.,  $f'(a)$ , is always (d)



- (a) the limit of  $f(x)$  as  $x$  approaches  $a$
- (b) the limit as  $h \rightarrow 0$  of the equation of the line joining the points  $(a, f(a))$  and  $(a + h, f(a + h))$
- (c) the limit of  $f'(x)$  as  $x \rightarrow a$
- (d) the limit as  $h \rightarrow 0$  of the slope of the line joining the points  $(a, f(a))$  and  $(a + h, f(a + h))$
- (e) none of the above

11. Which one of the following statements is false. (b)



- (a) If  $f'(x_0)$  exists then it is equal to the left sided limit  $\lim_{x \rightarrow x_0^-} \frac{f(x) - f(x_0)}{x - x_0}$ .
- (b) If  $|f(x)|$  is continuous at  $x = c$  then so is  $f(x)$ .
- (c)  $e^{\ln(x^4+1)} = x^4 + 1$ .
- (d) If  $\lim_{x \rightarrow 0} f(x) = L$  and  $f(x) > 0$  for all  $x$  in the interval  $(-1, 1)$  then  $L \geq 0$ .

**End of Examination**