UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 249 — L05&L07 FALL 2009

MIDTERM PRACTICE PROBLEMS

1. Let $y = x^2 \sin(3x) + \tan^2\left(\frac{1}{x}\right).$ Find y'.

2. Find the limits

$$\lim_{x\to-\infty} \frac{x + x^5 + 7}{3x - 4x^4 - 6x^5}$$

$$\lim_{x\to 0} \frac{1 - \cos 3x}{x^2}$$

$$\lim_{x\to\infty} \left[\sqrt{4x^2 - 3x + 1} - 2x\right]$$

$$\lim_{y\to 0} \frac{\tan(x+y) - \tan(x)}{y}$$

3. Find the equation of the tangent line to the graph of

$$y\cos(x+y) = x + y - \frac{\pi}{2}$$

at the point $\left(0, \frac{\pi}{2}\right)$.

4. Find all points on the graph of $y = x^2 + 2x$ where the tangent line also passes through the point $(3, -1)$.

5. Find the x coordinate of all points on the graph of $y = x^3 + x$ where the tangent line is parallel to the secant line which cuts the curve at $x = 1$ and $x = 3$.

6. Solve the inequality

$$\frac{3x+12}{x-3} \le \frac{2x-20}{x+5}$$

7. Let $f(x) = xe^{x\tan x}$. Find y'.

8. Let

$$y = \frac{1}{(\ln x)^2}$$

Find y'.

9. Functions f, g, and h are defined by

$$f(x) = 1 - \frac{1}{x^2}, \qquad g(x) = \sqrt{1 + \frac{1}{x}}, \qquad h = f \circ g.$$

Find $h(x)$ and simplify it.

Find the domains of f, g, and h. Express your answers in interval notation.

10. Let $f(x) = \sqrt{3x+10}$. Use the limit definition of derivative to find $f'(2)$.

11. Let $f(x) = x^2 - x\cos x$

    (a) Find the linear approximation to f at the point $x_0 = \dfrac{\pi}{2}$.

    (b) Suppose the linear approximation to f at $x_0 = \frac{\pi}{2}$ is used to find approximate values of $f(x)$. What is the error if it is used at $x = \pi$?

12. For $x < 1$ the graph of certain function $f(x)$ is the curve $y = 6x^{3/2} + x$. For $x \geq 1$ the graph of $f(x)$ is the line joining the points $(1, 7)$ and $(4, y_0)$.

    (a) Is f continuous at the point $x = 1$?

    (b) Find $\dfrac{f(1+h) - f(1)}{h}$ when $h < 0$.

    (c) Find $\displaystyle\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$.

    (d) Find a value of y_0 such that $f'(1)$ exists.