UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

# MATHEMATICS 249 — L07
# ANSWERS TO MIDTERM EXAM(a) NOV. 10, 2009

**Total Marks =80.** **Duration = 75 minutes.**
**Work all problems. Marks are shown in brackets.**
**NO CALCULATORS OR FORMULA SHEETS.**

[5] 1. Draw the graph of $\sin x$ for $0 \le x \le 2\pi$.

Locate all $x$ and $y$ intercepts on the graph and give their coordinates. Do the same for the point at $x = \pi/2$.

This graph was supposed to be drawn from memory.

See your lecture notes.

[3] 2. State the $\varepsilon - \delta$ definition of $\lim_{x \to a} f(x) = L$.

This question was supposed to be answered from memory.

See your lecture notes.

[9] 3. Find the slope of the tangent line to the graph of

$$2xy + y\cos y = \cos(x + y)$$

at the point $(0, \pi/2)$.

$$2y + 2xy' + y'\cos y - yy'\sin y = -\sin(x + y)(1 + y')$$

$$\pi - \frac{\pi}{2}y' = -(1 + y')$$

$$y' = -\frac{1 + \pi}{1 - \pi/2}$$

[10] 4. Let $f(x) = x\sin x + x\cos x$. Find the linear approximation to $f$ at the point $x_0 = \pi$.

$$f(x_0) = f(\pi) = -\pi$$

$$f'(x) = \sin x + x\cos x + \cos x - x\sin x \qquad f'(x_0) = f'(\pi) = -(\pi + 1)$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) = -\pi - (\pi + 1)(x - \pi)$$

5. For $x \le 1$ the graph of a certain function $f$ is the line joining the points $(-2, y_0)$ and $(1, 5)$. For $x > 1$ the graph of $f$ is the curve $y = 5x^{6/5}$.

[3] (a) Find $\dfrac{f(1+h) - f(1)}{h}$ when $h > 0$.

$$\frac{f(1+h) - f(1)}{h} = \frac{5(1+h)^{\frac{6}{5}} - 5}{h}$$

[3] (b) Find $\lim_{h\to 0^+} \dfrac{f(1+h)-f(1)}{h}$.

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = y'(1) = 6$$

[3] (c) Find a value of $y_0$ such that $f'(1)$ exists.

$$\frac{y_0-5}{-2-1} = 6 \qquad y_0 = -13$$

[9] 6. Find the limit

$$\lim_{h\to 0} \frac{e^{(2+h)}-e^2}{h}$$

$$\lim_{h\to 0} \frac{e^{(2+h)}-e^2}{h} = f'(2) = e^2 \qquad \textit{where} \quad f(x) = e^x \quad f'(x) = e^x$$

or use L'Hopital's rule

$$\lim_{h\to 0} \frac{\frac{d}{dh}[e^{(2+h)}-e^2]}{\frac{d}{dh}h} = \lim_{h\to 0} \frac{e^{(2+h)}-0}{1} = e^2$$

**Each of the following multiple choice questions has one and only one correct answer. Write the letter corresponding to your answer in the box provided. Each multiple choice question is worth 7 marks.**

7. Which one of the following limits is **NOT** $+\infty$ ? (d)

(a) $\lim_{x\to\infty} e^{3x}$

(b) $\lim_{x\to\infty} \ln x$

(c) $\lim_{x\to 0^+} \frac{\sin x}{x^2}$

(d) $\lim_{x\to 0^+} \ln x$

8. For

$$f(x) = x^4 \ln\left(\frac{1}{x} + 1\right)$$

$f'(x) =$ (d)

(a) $4x^3 \ln(\frac{-1}{x^2})$

(b) $\frac{-4x}{\frac{1}{x}+1}$

(c) $4x^3 \ln(\frac{1}{x} + 1) + \dfrac{x^4}{\frac{1}{x} + 1}$

(d) $4x^3 \ln(\frac{1}{x} + 1) - \dfrac{x^2}{\frac{1}{x} + 1}$

(e) none of the above

9. For a function $f$ the intermediate value theorem is (b)

(a) If $f$ is continuous on the open interval $(a, b)$ and $k$ is a number between $f(a)$ and $f(b)$ then there exists a point $c$ in $(a, b)$ for which $f(c) = k$.

(b) If $f$ is continuous on the closed interval $[a, b]$ and $k$ is a number between $f(a)$ and $f(b)$ then there exists a point $c$ in $(a, b)$ for which $f(c) = k$.

(c) If $k$ is a number between $f(a)$ and $f(b)$ then there exists a point $c$ in $(a, b)$ for which $f(c) = k$.

(d) If $f$ is continuous on the closed interval $[a, b]$ then the equation $f(x) = 0$ is satisfied by $x = c$ for some point $c$ in $(a, b)$.

(e) none of the above.

10. For every differentiable function $f$, its derivative at $x = a$, i.e., $f'(a)$, is always (a)

(a) the limit as $h \to 0$ of the slope of the line joining the points $(a, f(a))$ and $(a + h, f(a + h))$

(b) the limit of $f'(x)$ as $x \to a$

(c) the limit as $h \to 0$ of the equation of the line joining the points $(a, f(a))$ and $(a+h, f(a+h))$

(d) the limit of $f(x)$ as $x \to a$

(e) none of the above

11. Which one of the following statements is <u>false</u>. (c)

(a) If $f'(x_0)$ exists then it is equal to the left sided limit $\lim\limits_{x \to x_0^-} \dfrac{f(x) - f(x_0)}{x - x_0}$.

(b) The function

$$f(x) = \begin{Bmatrix} \sqrt{1-x^2} & 0 < x \leq 1 \\ -\sqrt{1-x^2} & -1 \leq x \leq 0 \end{Bmatrix}$$

is defined implicitly by the equation $x^2 + y^2 = 1$.

(c) $\ln(xe^x) = 1 + \ln x$.

(d) If $\lim\limits_{x \to 0} f(x) = L$ and $f(x) > 0$ for all $x$ in the interval $(-1, 1)$ then $L \geq 0$.

**End of Examination**