MATH 249 L05 and L07 FALL 2009 PRACTICE PROBLEMS FOR THE FINAL EXAM.

These problems relate mostly to the material since the midterm.

To cover the remainder of the course material use the set of practice problems for the midterm.

Some of these problems are similar to some of the problems on the final exam.

1. Find $\frac{dy}{dx}$.

(a) $y = \sin^3(\frac{1}{x})$.
(b) $y = x^3 \ln(3 + \sin(x^4))$.

(c) $y = (\frac{1}{x})^{\ln x}$.

(d) $y = \int_1^x \frac{\sin t}{t^2+1} dt$.

2. Find the limits.

(a) $\lim_{h \to 0} \frac{(1+h)^{\frac{1}{5}} - 1}{h}$

(b) $\lim_{x \to 0^+} (\sin x)(\ln x)$

3. State the mean value theorem and draw a picture to illustrate its meaning.

4. Find an equation of the straight line tangent to the curve $y = e^x$ and passing through the origin.

5. Given: $f(x) = \frac{x^3-6x^2+9x-3}{(x-1)^3}$, $\frac{df}{dx} = \frac{3x(x-2)}{(x-1)^4}$, $\frac{d^2f}{dx^2} = -\frac{6(x^2-2x-1}{(x-1)^5}$.
Find "everything" and the graph of f.

6. Suppose the velocity of a moving body is given as a function of the time t by $v(t) = \frac{e^t}{e^t+1}$. Suppose also that at $t = 0$ the position of the body, $s(t)$, has the initial value $s(0) = 3$.

(a) Find $s(t)$ and the acceleration $a(t)$.
(b) Find the average velocity and the average acceleration of this body between the times $t = 0$ and $t = 2$.

7. Let $f(x) = e^{2x} - 2e^x$.

(a) Find $\lim_{x\to\infty} f$ and $\lim_{x\to-\infty} f$.

(b) Sketch the graph of f. (Marks will be awarded only for the graph.)

8. Similar to 7 with $f(x) = x^2 \ln x$. Find the limits as $x \to \infty$ and as $x \to 0^+$ and find the graph of f.

9. Find the integrals.

(a) $\int(-1 + x^{-2} - x^{-1} + e^x)\, dx$

(b) $\int \frac{3x^5}{\sqrt{x^3+1}}\, dx$

(c) $\int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x dx$

(d) $\int_1^2 \frac{d}{dx}[x \ln(x^2 + x - 1)]dx$

10. Find the area bounded by the curve $y = x^{\frac{1}{3}}$, the x-axis, and the vertical lines $x = -1$ and $x = 8$.

11. Find the area enclosed by the graphs of $f(x) = \sqrt{x}$ and $g(x) = x^3$.

12. Let $f(x) = xe^{-x}$. Find the absolute maximum and absolute minimum of f over the interval $[-2, 2]$.