The University of Calgary
Department of Mathematics and Statistics
MATH 251 Quiz #3W FALL 2000

1. Using the definition of derivative find $f'(-1)$ if $f(x) = \dfrac{4x}{3-x}$. [3]

2. Find y' if $y = (\dfrac{x^6}{2} - 2x)(4 + \dfrac{1}{\sqrt{2x}})$ for $x > 0$. [3]

3. Find all points on the graph of $y = \dfrac{1}{2x^3 + x^2 + 1}$ where the tangent is horizontal. [4]

Solution

For 1)

first $f(-1) = \frac{-4}{4} = -1$

$$f'(-1) = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{\dfrac{4(-1+h)}{3-(-1+h)} - (-1)}{h} =$$

$$= \lim_{h\to 0} \frac{\dfrac{4h-4}{4-h} + 1}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{4h-4+4-h}{4-h}\right] =$$

$$= \lim_{h\to 0} \tfrac{1}{h} \cdot \frac{3h}{4-h} = \lim_{h\to 0} \frac{3}{4-h} = \frac{3}{4}$$

(Check by rules $f'(x) = \frac{4}{3-x} + \frac{4x}{(3-x)^2}$ at $x = -1$ $f'(-1) = 1 - \frac{4}{16} = \frac{3}{4}$)

also $\quad f'(-1) = \lim_{x\to -1} \dfrac{f(x) - f(-1)}{x - (-1)} = \lim_{x\to -1} \dfrac{\frac{4x}{3-x} + 1}{x+1} =$

$$= \lim_{x\to -1} \frac{\frac{4x+3-x}{3-x}}{x+1} = \lim_{x\to -1} \frac{\frac{3(x+1)}{3-x}}{x+1} = \lim_{x\to -1} \frac{3(x+1)}{(3-x)(x+1)} = \lim_{x\to -1} \frac{3}{3-x} = \frac{3}{4}$$

For 2)

use Product Rule $\quad y' = (\dfrac{x^6}{2} - 2x)'(4 + \dfrac{1}{\sqrt{2x}}) + (\dfrac{x^6}{2} - 2x)(4 + \dfrac{x^{-\frac{1}{2}}}{\sqrt{2}})' =$

now Power Rule $\quad y' = (\dfrac{1}{2}6x^5 - 2)(4 + \dfrac{1}{\sqrt{2x}}) + (\dfrac{x^6}{2} - 2x)(0 + \dfrac{1}{\sqrt{2}}\left(\dfrac{-1}{2}\right) x^{-\frac{3}{2}})$

so $\quad y' = (3x^5 - 2)(4 + \dfrac{1}{\sqrt{2x}}) + (\dfrac{x^6}{2} - 2x)(\dfrac{-1}{2\sqrt{2}} x^{-\frac{3}{2}})$ for $x > 0$

For 3)

slope of a tangent is given by $y' = \left(\dfrac{1}{2x^3 + x^2 + 1}\right)'$

we can use reciprocal (quotient) or Chain Rule

$$y' = \left([2x^3 + x^2 + 1]^{-1}\right)' = (-1)[2x^3 + x^2 + 1]^{-2} \cdot \cdot [2x^3 + x^2 + 1]' =$$

$$= \frac{(-1)[6x^2 + 2x + 0]}{[2x^3 + x^2 + 1]^2}$$

horizontal means the slope $m = 0$ solve for x $y' = 0$
a fraction is 0 only if top is 0 $6x^2 + 2x = 2x(3x+1) = 0$
thus at $x = 0, y = 1$ and at $x = -\frac{1}{3}, y = \frac{28}{27}$
two points $(0,1)\,, \left(-\frac{1}{3}, \frac{28}{27}\right)$ at which the tangent lines are horizontal