The University of Calgary Department of Mathematics and Statistics
MATH 249 Lecture 03 Quiz #4W

FALL 2002

Name:____________________________**I.D.#:**__________________

1. Find an equation of the tangent line to
   $2\sqrt{xy} = \frac{x}{y} + 3$ at the point $P(-2,-2)$. [4]

2. Find $\int \left(3\sqrt[4]{x} - \frac{x}{2}\right)^2 dx$ for $x \geq 0$. [3]

3. Solve $y'' = \sin(3x)$, $y(\frac{\pi}{2}) = \frac{1}{9}, y'(\frac{\pi}{2}) = 5$. [3]

SOLUTION
For 1)
an equation is $y = m(x+2) - 2$, where $m = y'$ at the point P.
for y'use implicit differentiation:
$2\left[(xy)^{\frac{1}{2}}\right]' = \left(\frac{y}{x}\right)' + (3)'$ by Chain and Quotient Rules
$2 \cdot \frac{1}{2}(xy)^{-\frac{1}{2}} \cdot (xy)' = \frac{y - xy'}{y^2} + 0$
$(xy)^{-\frac{1}{2}}(y + xy') = \frac{y - xy'}{y^2}$
now for $x = -2, y = -2, y' = m$ we get $\frac{1}{\sqrt{4}}(-2 - 2m) = \frac{-2 + 2m}{4}$
$-1 - m = \frac{1}{2}(-1 + m)$, $-2 - 2m = -1 + m$, $-1 = 3m$
$m = -\frac{1}{3}$ and an equation of the tangent is $y = \frac{-1}{3}(x+2) - 2$
.
For 2)
$\int \left(3\sqrt[4]{x} - \frac{x}{2}\right)^2 dx($ get rid of the power using $(A-B)^2 = A^2 - 2AB + B^2)$=
$= \int \left[\left(3x^{\frac{1}{4}}\right)^2 - 2 \cdot 3x^{\frac{1}{4}} \cdot \frac{x}{2} + \left(\frac{x}{2}\right)^2\right] dx =$
$= 9\int x^{\frac{1}{2}}dx - 3\int x^{\frac{5}{4}}d + \frac{1}{4}\int x^2 dx = 9 \cdot \left(\frac{2}{3}\right)x^{\frac{3}{2}} - 3 \cdot \frac{4}{9}x^{\frac{9}{4}} + \frac{1}{4} \cdot \frac{x^3}{3} + c = 6x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{9}{4}} + \frac{1}{12}x^3 + c$

For 3)$y'' = \sin(3x)$, $y(\frac{\pi}{2}) = \frac{1}{9}, y'(\frac{\pi}{2}) = 5$
$y' = \int y''dx = \int \sin(3x)dx = \left[\frac{-\cos 3x}{3}\right] + c_1 = -\frac{1}{3}\cos(3x) + c_1,$
now the condition $x = \frac{\pi}{2}, y' = 5$,solve for c_1 ;
$5 = -\frac{1}{3}\cos\left(\frac{3\pi}{2}\right) + c_1 = 0 + c_1 \qquad 5 = c_1,$
and $y' = -\frac{1}{3}\cos(3x) + 5$ for any x.

again
$y = \int y' dx = -\frac{1}{3} \int \cos(3x) dx + 5 \int dx + c_2 = -\frac{\sin 3x}{9} + 5x + c_2$

finally the condition $\quad y(\frac{\pi}{2}) = \frac{1}{9}$

$\frac{1}{9} = -\frac{1}{9} \sin \frac{3\pi}{2} + \frac{5\pi}{2} + c_2 = \frac{1}{9} + \frac{5\pi}{2} + c_2$ so $c_2 = \frac{-5\pi}{2}$

and the solution is $\quad y = -\frac{1}{9} \sin(3x) + 5x - \frac{5\pi}{2}$ for any x.