The University of Calgary **Department of Mathematics and Statistics**

MATH 249 **Lecture 01** **Quiz #4R**

FALL 2002

Name:____________________________**I.D.#:**_________________

1. Find an equation of the tangent line to
$\frac{x}{y} + 2\sin(xy) = 3\pi + \sqrt{3}$ at the point $P(\pi, \frac{1}{3})$. [4]

2. Find $\int \sqrt{x}\left(3\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{1}{3x}\right) dx$ for $x > 0$. [3]

3. Solve $y'' = 6\,(2x-3)^2\,, \quad y(1) = \frac{1}{8}, y'(1) = 2.$ [3]

SOLUTION

For 1)

an equation is $y = m(x-\pi) + \frac{1}{3}$, where $m = y'$ at the point P.

for y'use implicit differentiation:

$\left(\frac{x}{y}\right)' + 2\,[\sin(xy)]' = \left(3\pi + \sqrt{3}\right)'$ by Chain and Quotient Rules

$\frac{y - xy'}{y^2} + 2\cos(xy)\,(xy)' = 0$ byProduct Rule

$\frac{y - xy'}{y^2} + 2\cos(xy)\,(y + xy') = 0$

now for $x = \pi, y = \frac{1}{3}, y' = m$ we get

$9\left(\frac{1}{3} - \pi m\right) + 2\cos\frac{\pi}{3}\left(\frac{1}{3} + \pi m\right) = 0 \qquad \left(\cos\frac{\pi}{3} = \frac{1}{2}\right)$

$3 - 9\pi m + \frac{1}{3} + \pi m = 0, \quad \frac{10}{3} = 8\pi m, \quad m = \frac{5}{12\pi}$

and an equation of the tangent is $\quad y = \frac{5}{12\pi}(x-\pi) + \frac{1}{3}.$

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For 2)

$\int \sqrt{x}\left(3\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{1}{3x}\right) dx$(get rid of the product)

$= \int \left[3\sqrt{x}\sqrt{x} + \sqrt{x}\cdot\frac{2}{\sqrt{x}} - \frac{\sqrt{x}}{3x}\right] dx =$

$= 3\int x dx + 2\int dx - \frac{1}{3}\int x^{-\frac{1}{2}} dx = \frac{3}{2}x^2 + 2x - \frac{2}{3}x^{\frac{1}{2}} + c.$

For 3)

$y'' = 6\,(2x-3)^2\,, \quad y(1) = \frac{1}{8}, y'(1) = 2$

$y' = \int y'' dx = 6\int (2x-3)^2\,dx = 6\cdot\frac{(2x-3)^3}{3\cdot 2} + c_1 = (2x-3)^3 + c_1,$

now the condition $x = 1, y' = 2$,solve for c_1 ;

$2 = (-1)^3 + c_1 = -1 + c_1 \qquad 3 = c_1,$

and $\qquad y' = (2x-3)^3 + 3 \qquad$ for any x.

again

$y = \int y' dx = \int (2x-3)^3 dx + 3\int dx + c_2 = \dfrac{(2x-3)^4}{4 \cdot 2} + 3x + c_2$

finally the condition $\quad y(1) = \dfrac{1}{8}$

$\dfrac{1}{8} = \dfrac{1}{8}(-1)^4 + 3 + c_2 = \dfrac{1}{8} + 3 + c_2$ so $c_2 = -3$

and the solution is $\qquad y = \dfrac{1}{8}(2x-3)^4 + 3x - 3 \qquad$ for any x.

Note: you can also simplify first

$y'' = 6\,(2x-3)^2 = 24x^2 - 72x + 54$ and then integrate.