**The University of Calgary**
**Department of Mathematics and Statistics**
**MATH 249-01 Quiz #3W FALL 2003**

1. Using the definition of derivative find $f'(-1)$ if $f(x) = \sqrt{12+3x}$. [3]

2. Using Differentiation Rules find $f'$ if $f(x) = \dfrac{2-x}{\sqrt{x^2+1}}$ [3]

3. Find an equation of the tangent to $y = \left(\dfrac{x^2}{2} - 3x\right)\left(\dfrac{3}{x}+1\right)$ at $x = 3$. [4]

**Solution**

**For 1)**

first $f(-1) = \sqrt{12+3x} = 3$ then

$$f'(-1) = \lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1}\frac{\sqrt{12+3x}-3}{x+1} = \lim_{x\to -1}\frac{\sqrt{12+3x}-3}{x+1}\cdot\frac{\sqrt{12+3x}+3}{\sqrt{12+3x}+3}$$

$$= \lim_{x\to -1}\frac{(12+3x)-9}{(x+1)\left(3+\sqrt{12+3x}\right)} =$$

$$= \lim_{x\to -1}\frac{3(x+1)}{(x+1)\left(3+\sqrt{12+3x}\right)} = \lim_{x\to -1}\frac{3}{\left(3+\sqrt{12+3x}\right)} = \frac{3}{6} = \frac{1}{2}$$

check by Chain Rule:

$f'(x) = \left[(12+3x)^{\frac{1}{2}}\right]' = \left(\dfrac{1}{2}\right)(12+3x)^{-\frac{1}{2}}(3)$ and at $x=-1$ we get $\dfrac{3}{2\cdot 3} = \dfrac{1}{2}$

**For 2)**

use Quotient and Chain Rules

$$f'(x) = \frac{(2-x)'\sqrt{x^2+1} - (2-x)\left(\sqrt{x^2+1}\right)'}{\left(\sqrt{x^2+1}\right)^2} = \frac{(-1)\sqrt{x^2+1} + (x-2)\cdot\dfrac{2x}{2\sqrt{x^2+1}}}{x^2+1}$$

we can simplify

$$f'(x) = \frac{-(x^2+1)+x^2-2x}{(x^2+1)\sqrt{x^2+1}} = \frac{-1-2x}{(x^2+1)\sqrt{x^2+1}}$$

**For 3)**

the point is $x=3$ and $y = \left(\dfrac{9}{2}-9\right)\left(\dfrac{3}{3}+1\right) = -9$ $\quad P(3,-9)$

so an equation of the tangent is $y = m_t(x-3) - 9$

to find the slope you may multiply out first

$y = \dfrac{3}{2}x - 9 + \dfrac{1}{2}x^2 - 3x$ then $y' = \dfrac{3}{2} + x - 3 = -\dfrac{3}{2} + x$

and at $x=3$ $\quad m_t = \dfrac{3}{2}$

Or use Product Rule

$$y' = \left(\frac{1}{2}x^2 - 3x\right)'\left(\frac{3}{x} + 1\right) + \left(\frac{x^2}{2} - 3x\right)(3x^{-1} + 1)' =$$

$$= \left(\frac{1}{2} \cdot 2x - 3\right)\left(\frac{3}{x} + 1\right) + \left(\frac{x^2}{2} - 3x\right)(-3x^{-2})$$

at $x = 3$ $\qquad m_t = 0 + \left(\frac{9}{2} - 9\right)\frac{-3}{3^2} = \frac{3}{2}$ $\qquad$ FINALLY

$$y = \frac{3}{2}(x - 3) - 9 \qquad \text{OR} \qquad 3x - 2y = 27.$$