The University of Calgary
Department of Mathematics and Statistics
MATH 249//03
Quiz # 4W

FALL 2005

Name:_____**I.D.#:**_____

1. Find an equation of the tangent to

$$xy^2 + y = \frac{2x^3}{y} - 1$$

at the point $(-1, 2)$. [4]

2. Find a general antiderivative of $f(x) = \dfrac{4\sqrt{x} - x^3 - 2}{2x^2}$ for $x > 0$. [3]

3. Solve $y' = 2\cos(\frac{x}{2} + \pi)$, $y(-\frac{\pi}{2}) = 0$.. [3]

Solution For1)

an equation is $y = m(x+1) + 2$ for m use the implicit differentiation

$(xy^2)' + (y)' = 2\left(\dfrac{x^3}{y}\right)' - 0$ product and quotient rules

$y^2 + x2yy' + y' = 2 \cdot \dfrac{3x^2y - x^3y'}{y^2}$ now $x = -1, y = 2, y' = m$

$4 - 4m + m = 2 \cdot \dfrac{6+m}{4}$ $4 - 3m = 3 + \dfrac{1}{2}m$ $1 = \dfrac{7}{2}m$

so $m = \dfrac{2}{7}$ and an equation is $y = \frac{2}{7}(x+1) + 2$ or $7y - 2x = 16$

For 2)

$\int \dfrac{4\sqrt{x} - x^3 - 2}{2x^2} dx = 2\int \dfrac{\sqrt{x}}{x^2} dx - \frac{1}{2}\int \dfrac{x^3}{x^2} dx - \int \dfrac{1}{x^2} dx =$

$= 2\int x^{-\frac{3}{2}} dx - \frac{1}{2}\int x dx - \int x^{-2} dx = -4x^{-\frac{1}{2}} - \dfrac{1}{4}x^2 + x^{-1} + c, x > 0.$

For 3)

$y = \int y' dx = 2\int \cos(\frac{x}{2} + \pi) dx = 4\sin\left(\dfrac{x}{2} + \pi\right) + c$

using $\int \cos(ax+b) dx = \frac{1}{a}\sin(ax+b) + c$ where $a = \frac{1}{2}$

now, $y = 0, x = -\frac{\pi}{2}, c = ?$

$0 = 4\sin\left(-\dfrac{\pi}{4} + \pi\right) + c = 4\sin\left(\dfrac{3\pi}{4}\right) + c = 2\sqrt{2} + c$ and $c = -2\sqrt{2}$ finally,

$y = 4\sin\left(\dfrac{x}{2} + \pi\right) - 2\sqrt{2}$

ALSO, $\cos(\frac{x}{2} + \pi) = -\cos\dfrac{x}{2}$ so $y = -2\int \cos(\frac{x}{2}) dx = -4\sin\dfrac{x}{2} + c$