The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz #5R Fall 2006

1. Solve for x : $2\ln(x+1) - \ln(4x) = 0.$ [3]

2. For $f(x) = 2^{\sqrt{x}} + (\frac{1}{x})^x$ find the domain and $f'(x)$. [4]

3. How long does it take to double your investement if the annual interest is 2.5% compounded monthly? [3]

SOLUTION

For 1)

simplify first $\ln(x+1)^2 = \ln(4x)$ then apply exp. f.

$(x+1)^2 = 4x \qquad x^2 - 2x + 1 = 4x$

$x^2 - 2x + 1 = 0 \qquad (x-1)^2 = 0$

$x = 1$ is the solution.

For 2)

it must $x > 0$ rewrite $(\frac{1}{x})^x = e^{x \ln \frac{1}{x}} = e^{-x \ln x}$

$$f'(x) = \left(2^{\sqrt{x}}\right)' + (e^{-x\ln x})' = 2^{\sqrt{x}} \ln 2 \left(\sqrt{x}\right)' + e^{-x\ln x} \left(-x \ln x\right)' =$$

$$= \frac{\ln 2}{2\sqrt{x}} 2^{\sqrt{x}} - e^{-x\ln x} \left(\ln x + 1\right)$$

or log.diif BUT only for $u = (\frac{1}{x})^x \qquad \ln u = \ln\,(\frac{1}{x})^x = -x\ln x$

then $\dfrac{u'}{u} = -\ln x - x \cdot \frac{1}{x} \qquad u' = u\,(-1)\,(\ln x + 1) = -(\frac{1}{x})^x\,(\ln x + 1)$

For 3)

the correct formula for the amount of money is $A(t) = A_0 \left(1 + \frac{p}{100n}\right)^{nt}$

t in years ,the initial amount A_0 should change to $2A_0 \qquad p = 2.5 \qquad n = 12$

thus $2A_0 = A_0 \left(1 + \frac{2.5}{1200}\right)^{12t} \qquad 2 = \left(\frac{12025}{12000}\right)^{12t}$ solve for t by applying ln

$\ln 2 = 12t \cdot \ln \frac{12025}{12000} \qquad t = \frac{\ln 2}{12 \ln \frac{12025}{12000}} = 27.755$ years