The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz #5W Fall 2006

1. Solve for x : $\dfrac{1}{3^{2x+1}} = \dfrac{5}{9^{3x}}$. [3]

2. For $f(x) = x^{\sqrt{x}} + \ln(2x-1)$ find the domain and $f'(x)$. [4]

3. After 3 days a sample of radon decayed to 45 kg.After one more day to 35 kg. What was the original amount? [3]

SOLUTION **For 1)**

apply ln to both sides $\dfrac{1}{3^{2x+1}} = \dfrac{5}{9^{3x}}$

$\ln 1 - (2x+1)\ln 3 = \ln 5 - 3x \ln 9$

$3x \ln 3^2 - 2x \ln 3 = (6x - 2x)\ln 3 = \ln 5 + \ln 3 = \ln 15$

$4x ln 3 = ln 15$

or $x\,(3\ln 9 - 2\ln 3) = x \ln \frac{3^6}{3^2}$

and finally $x \ln 3^4 = \ln 15$ $x = \dfrac{\ln 15}{\ln 81} = \dfrac{\ln 15}{4 \ln 3}$

OR cross multiply first $\dfrac{9^{3x}}{3^{2x+1}} = \dfrac{3^{6x}}{3^{2x+1}} = 3^{4x-1} = 5$ $3^{4x} = 15$

and then log.f. $4x = \log_3 15$ $x = \frac{1}{4}\log_3 15$

For 2)

it must $x > 0$ and $2x - 1 > 0$ so $x > \frac{1}{2}$ rewrite $x^{\sqrt{x}} = e^{\sqrt{x}\ln x}$

$f'(x) = \left(e^{\sqrt{x}\ln x}\right)' + (\ln(2x-1))' = e^{\sqrt{x}\ln x}\left(\sqrt{x}\ln x\right)' + \dfrac{1}{2x-1}\cdot 2 =$

$= e^{\sqrt{x}\ln x}\left(\dfrac{1}{2\sqrt{x}}\cdot \ln x + \dfrac{\sqrt{x}}{x}\right) + \dfrac{2}{2x-1} = e^{\sqrt{x}\ln x}\left(\dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}\right) + \dfrac{2}{2x-1}$

or log.diff BUT only for $u = x^{\sqrt{x}}$ $\ln u = \ln\, x^{\sqrt{x}} = \sqrt{x}\ln x$

then $\dfrac{u'}{u} = \dfrac{\ln x}{2\sqrt{x}} + \dfrac{x}{\sqrt{x}}$ $u' = u\left(\dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}\right) = x^{\sqrt{x}}\left(\dfrac{\ln x}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}\right)$

For 3)

the correct formula for the amount of radon $A(t) = A_0 e^{kt}$ where $k < 0$

t in days ,the initial amount $A_0 =?$
given if $t = 3$ $45 = A_0 e^{3k}$ if $t = 4$ $35 = A_0 e^{4k}$
so $A_0 = \dfrac{45}{e^{3k}} = \dfrac{35}{e^{4k}}$ first solve for k $e^{4k-3k} = \frac{35}{45}$ $e^k = \dfrac{7}{9}$
$k = \ln \frac{7}{9} = -0.2513144$ and $A_0 = \dfrac{45}{e^{3k}} = 45 \cdot \frac{9^3}{7^3} = 95.64 kg$