

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249-01 Quiz #3W FALL 2007

1. Using the **definition of derivative** find $f'(-1)$ if $f(x) = \frac{2x}{3+x}$. [5]

2. Find y' if $y = \sqrt{x^2+1}(4 + \frac{1}{2x})$ for $x \neq 0$. [5]

3. Find an equation of the tangent line to $y = \frac{2\sqrt{x}}{2-x}$ at $x = 4$. [5]

Solution

For 1)

$$f'(-1) = \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \rightarrow -1} \frac{\frac{2x}{3+x} - \frac{-2}{2}}{x + 1} = \lim_{x \rightarrow -1} \frac{\frac{2x}{3+x} + 1}{x + 1} =$$

$$= \lim_{x \rightarrow -1} \frac{\frac{2x+3+x}{3+x}}{x + 1} = \lim_{x \rightarrow -1} \frac{\frac{3(x+1)}{3+x}}{x + 1} = \lim_{x \rightarrow -1} \frac{3}{x + 3} = \frac{3}{2}$$

$$f'(-1) = \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2(-1+h)}{3+(-1+h)} + 1}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2h-2}{2+h} + 1}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{2h-2+2+h}{2+h} \right] =$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \cdot \frac{3h}{2+h} = \lim_{h \rightarrow 0} \frac{3}{2+h} = \frac{3}{2}$$

(Check by rules $f'(x) = 2 \cdot \frac{3+x-x}{(3+x)^2} = \frac{6}{(3+x)^2}$ at $x = -1$ $f'(-1) = \frac{6}{4} = \frac{3}{2}$)

For 2)

use Product and Chain Rules for $x \neq 0$

$$y = \left[(x^2+1)^{\frac{1}{2}} \right]' \left(4 + \frac{1}{2x} \right) + \sqrt{x^2+1} \left(4 + \frac{1}{2}x^{-1} \right)' =$$

$$= \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \cdot (x^2+1)' \left(4 + \frac{1}{2x} \right) + \sqrt{x^2+1} \left(0 + \frac{1}{2} (-1) x^{-2} \right) =$$

$$\text{so } y' = \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \cdot 2x \left(4 + \frac{1}{2x} \right) + \sqrt{x^2+1} \left(\frac{-1}{2x^2} \right) = \frac{4x + \frac{1}{2}}{\sqrt{x^2+1}} - \frac{\sqrt{x^2+1}}{2x^2}$$

For 3)

at $x = 4$ $y = \frac{4}{-2} = -2$ the point is $P(4, -2)$ and the line through P is $y = -2 + m(x - 4)$

the slope of a tangent is given by $y' = \left(\frac{2\sqrt{x}}{2-x} \right)'$ at $x = 4$

we can use Quotient Rule

$$y' = \frac{(2\sqrt{x})'(2-x) - 2\sqrt{x}(2-x)'}{(2-x)^2} = \frac{2 \cdot \frac{1}{2\sqrt{x}}(2-x) - 2\sqrt{x}(-1)}{(2-x)^2}$$

at $x = 4$

$$\begin{aligned} m &= \frac{\frac{1}{\sqrt{4}}(-2) + 2\sqrt{4}}{(-2)^2} = \frac{3}{4} & y &= -2 + \frac{3}{4}(x - 4) \\ \text{or } y &= \frac{3}{4}x - 5 & \text{or } 3x - 4y &= 20 \end{aligned}$$