The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz #3R

FALL 2008

Name:______________________________**I.D.#:**__________________]

1. Using the **definition of the derivative** find $f'(-4)$ if $f(x)\sqrt{1-2x}.$ [4]

2. Find $f'(x)$ if $f(x)=\dfrac{\sqrt{4x}}{x^2+5}$ at $x=1.$ [3]

3. Find all points on the graph $y=\left(\dfrac{x^2}{3}-3\right)^3$ with horizontal tangents. [3]

For 1) $f(-4)=\sqrt{1+8}=3$

$$f'(-4)=\lim_{x\to-4}\frac{f(x)-f(-4)}{x+4}=\lim_{x\to-4}\frac{\sqrt{1-2x}-3}{x+4}\cdot\frac{\sqrt{1-2x}+3}{\sqrt{1-2x}+3}=$$

$$\lim_{x\to-4}\frac{1-2x-3^2}{x+4}\cdot\frac{1}{\sqrt{1-2x}+3}=\lim_{x\to-4}\frac{-2(x+4)}{x+4}\cdot\frac{1}{\sqrt{1-2x}+3}=$$

$$=\lim_{x\to-4}\frac{-2}{\sqrt{1-2x}+3}=\frac{-2}{6}=-\frac{1}{3}$$

OR

$$f'(-4)=\lim_{h\to0}\frac{f(-4+h)-f(-4)}{h}=\lim_{h\to0}\frac{\sqrt{1-2(-4+h)}-3}{h}=$$

$$\lim_{h\to0}=\frac{\sqrt{9-2h}-3}{h}\cdot\frac{\sqrt{9-2h}+3}{\sqrt{9-2h}+3}=\lim_{h\to0}=\frac{9-2h-3^2}{h}\cdot\frac{1}{\sqrt{9-2h}+3}$$

$$=\lim_{h\to0}=\frac{-2h}{h}\cdot\frac{1}{\sqrt{9-2h}+3}=\frac{-2}{3+3}=-\frac{1}{3}.$$

Check by Rules: $f'(x)=\frac{1}{2}(1-2x)^{-\frac{1}{2}}\cdot(-2)$ and at $x=-4$,

$$f'(-4)=\frac{-1}{\sqrt{9}}=-\frac{1}{3}.$$

For 2) Simplify first $\sqrt{4x}=\sqrt{4}\cdot\sqrt{x}=2\sqrt{x}$

then for $x>0$ by Quotient Rule

$$f'(x)=\left(\frac{\sqrt{4x}}{x^2+5}\right)'=\left(\frac{2x^{\frac{1}{2}}}{x^2+5}\right)'=\frac{\left(2x^{\frac{1}{2}}\right)'(x^2+5)-2\sqrt{x}\,(x^2+5)'}{(x^2+5)^2}=$$

$$.=.\frac{2\cdot\frac{1}{2}x^{-\frac{1}{2}}(x^2+5)-2\sqrt{x}\cdot2x}{(x^2+5)^2}=\frac{\frac{1}{\sqrt{x}}(x^2+5)-4x\sqrt{x}}{(x^2+5)^2}..$$

at $x=1$

$$f'(1)=\frac{6-4}{36}=\frac{1}{18}$$

Or

if not simplified you have to use Chain Rule

$$\left(\sqrt{4x}\right)' = \frac{1}{2}(4x)^{-\frac{1}{2}} \cdot (4x)' = \frac{1}{2}(4x)^{-\frac{1}{2}} \cdot 4 = \frac{2}{\sqrt{4x}}$$

$$f'(x) = \left(\frac{\sqrt{4x}}{x^2+5}\right)' = \frac{\left(\sqrt{4x}\right)'(x^2+5) - \sqrt{4x}\,(x^2+5)'}{(x^2+5)^2} =$$

$$. = .\frac{\frac{2}{\sqrt{4x}}(x^2+5) - \sqrt{4x} \cdot 2x}{(x^2+5)^2} \text{ and at } x = 1, f'(1) = \frac{\frac{2}{2} \cdot 6 - 2 \cdot 2}{36} = \frac{1}{18}.$$

For 3)

horizontal tangent means $m = y' = 0$, by Chain Rule

$$y' = \left[\left(\frac{x^2}{3} - 3\right)^3\right]' = 3\left(\frac{x^2}{3} - 3\right)^2 \cdot \left(\frac{1}{3}x^2 - 3\right)' = 3\left(\frac{x^2}{3} - 3\right)^2 \cdot \frac{1}{3} \cdot 2x = 2x\left(\frac{1}{3}x^2 - 3\right)^2$$

solve for x when $y' = 0$

For $x = 0, y = (-3)^3 = -27$,so one point is $P(0, -27)$

for $\frac{x^2}{3} - 3 = 0 \to x^2 = 9$ $\quad x = \pm 3, y = \left(\frac{9}{3} - 3\right)^3 = 0$ two more points $(\pm 3, 0)$.