

THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL Handout
MATH 249-01

1. Evaluate the limits:

$$(a) \quad \lim_{x \rightarrow \pi} \frac{\cos\left(\frac{x}{2}\right)}{\pi - x} \quad (b) \quad \lim_{x \rightarrow -\infty} \frac{\cos\left(\frac{x}{2}\right)}{\pi - x} \quad (c) \quad \lim_{x \rightarrow 0^+} x \ln x$$

2. Find the domain and the derivative of f of

$$(a) \quad f(x) = \frac{x}{3} e^{-\sin\left(\frac{3}{x}\right)}$$

$$(b) \quad f(x) = \frac{\ln(2x - 3)}{e^{-x^2}}$$

3. **A**

Sketch the graph of $y = \frac{1}{x^2 + x - 2}$ if $y' = -\frac{2x + 1}{(x^2 + x - 2)^2}$ and $y'' = \frac{6x^2 + 6x + 6}{(x^2 + x - 2)^3}$ i.e.

- (a) find the domain, range, vertical and horizontal asymptotes, x and y intercepts;
- (b) find the intervals where f is increasing or decreasing;
- (c) find the intervals where f is concave down or up

3B

Sketch the graph of $y = x(4 - x)^3$. Indicate where the function is increasing, decreasing, concave up, concave down; find the domain and range.

4. (a) Find the tangent approximation (linearization) of

$$f(x) = \frac{1}{\sqrt{2x^2 + 1}} \text{ around } x_0 = 2.$$

(b) Use it to estimate $\frac{1}{\sqrt{3}}$.

5. **A**

Sketch a graph of one function f satisfying all the following conditions:

- (a) f is defined for all x , continuous there except
- (b) f is discontinuous at $x = 2, 4$ where $\lim_{x \rightarrow 4^-} f(x) = f(4) = 0$, $x = 2$ is a vertical asymptote.
- (c) $y = 3$ is a horizontal asymptote and $\lim_{x \rightarrow -\infty} f(x)$ does not exist,

- (d) f is increasing on $(3, 4)$ and on $(4, +\infty)$, f is decreasing on $(0, 2)$ and on $(2, 3)$, and $f'(x) = 0$ for all $x \in (-2, 0)$;
- (e) f is concave up on $(0, 1)$ and on $(3, 4)$; f is concave down on $(1, 2)$, on $(2, 3)$ and on $(4, +\infty)$
- (f) absolute maximum value is 6, and local minimum value is -2 .

B

Sketch a graph of one function f satisfying all the following conditions:

- (a) f is defined on $(0, +\infty)$, continuous there except
- (b) f is discontinuous at $x = 1, 2, 3$ where $\lim_{x \rightarrow 2} f(x) = 3$, $\lim_{x \rightarrow 3} f(x)$ DNE (does not exist)
- (c) $x = 1$ is a vertical asymptote; $y = 2$ is a horizontal asymptote;
- (d) f is increasing on $(2, 3)$, $(3, 4)$ and on $(5, +\infty)$; f is decreasing on $(0, 1)$ and on $(4, 5)$
and $f'(x) = 0$ for all $x \in (1, 2)$;
- (e) f is concave up on $(2, 3)$ and on $(4, 6)$;
 f is concave down on $(6, +\infty)$ and on $(3, 4)$;
- (f) absolute maximum value is 5, and local minimum value is -1 .

C

Sketch a graph of one function f satisfying all the following conditions:

- (a) f is defined on $[-1, +\infty)$ and continuous there except ;
- (b) f is discontinuous at $x = 1, 3$ where $\lim_{x \rightarrow 1^-} f(x)$ DNE;
- (c) $x = 3$ is a vertical asymptote; $y = 2$ is a horizontal asymptote;
- (d) f is increasing on $(-1, 0)$ and on $(3, +\infty)$, f is decreasing on $(0, 1)$ and on $(2, 3)$, and $f'(x) = 0$ for all $x \in (1, 2)$;
- (e) f is concave up on $(-1, 0)$, $(3, 4)$ and on $(0, 1)$; f is concave down on $(2, 3)$ and on $(4, +\infty)$
- (f) absolute maximum value is 7, and local minimum value is 0.;

6. A

A box with a square base(bottom) and a top(lid) has a volume of 18 m^3 . Find the dimensions of the most economical box if the material

for the base and lid costs $\$2$ per m^2 and the material for the sides $\$3$ per m^2 .

B

A landscape architect plans to enclose a 280 m^2 rectangular region in a botanical garden.

She will use shrubs costing \$25.00 per meter along three sides and fencing costing \$10.00 per meter along the fourth side.

Find the dimensions of the region to minimize the total cost.

7. Find (a) for $x > 0$ $\int \frac{3\sqrt{x} - 5}{x\sqrt{x}} dx =$ (b) $\int 2x^3\sqrt{2x^2 + 3} dx.$

8. Evaluate (a) $\int_{\frac{1}{2}}^1 \frac{3^{\frac{1}{x}}}{x^2} dx$ (b) $\int_0^1 \frac{4x + 3}{3 - 2x} dx.$

9. (a) Solve for x : $2 \ln(x + 1) - \ln(4x) = 0.$

(b) Solve for x : $\frac{1}{2^{2x+1}} = \frac{5}{4^{3x}}.$