FINAL EXAMINATION **Math 249 L(01) - Fall, 2006** Time: 2 hours

NOTE: A calculator *is* allowed.

1. Find $(a)\lim\limits_{x\to 0}\dfrac{1-\cos x}{e^{x^2}-1} = "\frac{0}{0}"$ (L'H.R.)$=\lim\limits_{x\to 0}\dfrac{\sin x}{2xe^{x^2}} = \dfrac{1}{2e^0}\lim\limits_{x\to 0}\dfrac{\sin x}{x} = \dfrac{1}{2};$

   $(b)\lim\limits_{x\to+\infty}\dfrac{1-\cos x}{e^{x^2}-1} = "\frac{DNE}{\infty}" = 0$

   by Sq.Th.since $-1 \le -\cos x \le 1 \qquad 0 \le 1-\cos x \le 2$ so

   $0 \le \dfrac{1-\cos x}{e^{x^2}-1} \le \dfrac{2}{e^{x^2}-1}$ and $"\frac{2}{\infty}" = 0;$

   $(c)\lim\limits_{x\to\infty}\dfrac{\ln x}{e^{-x^2}-1} = "\dfrac{\infty}{-1}" = -\infty$ since "$e^{-\infty}$" $= 0$ (NO L'H.R.)

2. for the domain in (a) $3x+2>0$ and $2-x>0 \to -\frac{2}{3}<x<2$

   $D=\left(-\frac{2}{3},2\right)$ and $f'(x)=\left(\dfrac{\ln(3x+2)}{\sqrt{2-x}}\right)' = \dfrac{\frac{3}{3x+2}\sqrt{2-x}-\ln(3x+2)\frac{-1}{2\sqrt{2-x}}}{2-x} =$

   $= \dfrac{6(2-x)+(3x+2)\ln(3x+2)}{2\,(3x+2)\,(2-x)\,\sqrt{2-x}};$

   (b) for $x>0 \qquad g'(x)=\left(x^{\cos x}\right)' = e^{\cos x\ln x}.\,(\cos x\cdot\ln x)' = e^{\cos x\ln x}.\,\left(-\sin x\cdot\ln x+\frac{\cos x}{x}\right)$

   or by log.diff. $\ln g(x) = (\cos x\cdot\ln x) \qquad \frac{g'}{g} = \left(-\sin x\cdot\ln x+\frac{\cos x}{x}\right)$ then as above.

3. Find a tangent line approximation= linearization of $f(x)=\sqrt[3]{13-5x}$ around $x_0=1$; then use it to estimate $\sqrt[3]{9}$.

   $f(1)=\sqrt[3]{8}=2 \qquad f'(x)=\frac{1}{3}\,(13-5x)^{-\frac{2}{3}}\,(-5) = \dfrac{-5}{3\,(13-5x)^{\frac{2}{3}}},\ f'(1)=\frac{-5}{3\cdot\left(\sqrt[3]{8}\right)^2} = -\dfrac{5}{12}$

   tangent line: $y=2-\frac{5}{12}(x-1)$ then $\sqrt[3]{13-5x} \doteq L(x)=2-\frac{5}{12}(x-1)$ around $x=1$

   we need $\sqrt[3]{9}=\sqrt[3]{13-5x}$ so $9=13-5x \qquad x=\dfrac{4}{5}$ substitute into approximation equation

   $f(\frac{4}{5}) \doteq L(\frac{4}{5}) \qquad \sqrt[3]{9} \doteq 2-\frac{5}{12}\left(-\frac{1}{5}\right) = 2+\frac{1}{12} = 2.08333.$

4. A rectangular box with a square base and a square lid is to hold 12cm^3.

   Find the dimensions of the most economical box if the material for the base costs 4 cents per cm^2,

   and the material for the sides and lid costs 2 cents per cm^2.

let's call the dimensions of the base $x\times x$ and the height y, then the volum $V=x^2y=12$

thus $\qquad y=\dfrac{12}{x^2}$

the cost $\qquad C=4x^2+2(x^2+4xy)$ together $f(x)=4x^2+2(x^2+4x\frac{12}{x^2})=6x^2+\dfrac{8\cdot 12}{x}, x>0$

now for critical points $f'(x)=12x-\frac{8\cdot12}{x^2}=12\cdot\frac{x^3-8}{x^2}=0 \qquad x=2 \qquad y=\frac{12}{4}=3$

$f''(x)=12+\frac{8\cdot24}{x^3}>0$ at $x=2$ so C.P. is a minimum,and dimensions are $2\times2\times3\ cm.$

5. For $f(x)=\dfrac{x^2}{x-4}$ find

   (a) the domain, vertical and horizontal asymptotes:$\{\ x\neq4\}=D$,

   $x=4$ is V.A.since $\displaystyle\lim_{x\to+4^+}\frac{x^2}{x-4}=+\infty$ and $\displaystyle\lim_{x\to+4^-}\frac{x^2}{x-4}=-\infty$;

   NO H.A. since $\displaystyle\lim_{x\to++\infty}\frac{x^2}{x-4}=+\infty$ and $\displaystyle\lim_{x\to+-\infty}\frac{x^2}{x-4}=-\infty$

   (b) intervals where f is increasing, resp. decreasing:

   $f'(x)=\left(\dfrac{x^2}{x-4}\right)'=\dfrac{2x(x-4)-x^2}{(x-4)^2}=\dfrac{x^2-8x}{(x-4)^2}=\dfrac{x(x-4)}{(x-4)^2}$

   $x=4$ is a sing. point;$x=0,8$ are Crit.points

   testing $\qquad f'\ {-}^{pos}---_0--^{neg}---_4--^{neg}---_8-^{pos}--$

   thus $\qquad f$ is incr. on $(-\infty,0)$ and on $(8,+\infty)$; it is decr.on $(0,4)$ and $(4,8)$

   $f(0)=0 \qquad f(8)=16$

   (c) intervals where f is concave up resp. down: $f''(x)=\dfrac{32}{(x-4)^3}$

   $x=4$ is a sing. point;

   testing $\qquad f''\ {-}^{neg}---_4--^{pos}---$

   thus f is conc.up on$(4,\infty)$ and conc. down on $(-\infty,4)$

   (d) all local and absolute extrema, and the range: from b) or c)$x=0,y=0$ is loc. max

   and$x=8$,$y=16$ is a local minb and we have a gap in the range $R=(-\infty,0]\cup[16,\infty)$.

6. graph

7. For (a) $\qquad\displaystyle\int x\sqrt{3x^2+2}dx=(\text{by subst.}u=3x^2+2,du=6dx)=\frac{1}{6}\int u^{\frac12}du=$

   $=\frac16\cdot\frac23u^{\frac32}+c=\frac19\left(3x^2+2\right)^{\frac32}+c.$

   (b) $\qquad\displaystyle\int\cos x\sin^2x\ dx=(\text{ by subst.}u=\sin x,du=\cos xdx)=\int u^2du=\frac13\sin^3x+c.$

8. For(a) by subst., $u = 3x+1, du = 3dx,$ $\begin{array}{cc} x & u \\ 1 & 4 \\ 2 & 7 \end{array}$ and $3x = u - 1$ $3x + 1 = u$

$$\int_1^2 \frac{6x}{3x+1} dx = \tfrac{2}{3} \int_4^7 \frac{u-1}{u} du = \tfrac{2}{3} \int_4^7 \left[1 - \frac{1}{u} \right] du = \tfrac{2}{3} \left[u - \ln |u| \right]_4^7 = 2 - \tfrac{2}{3} \ln \tfrac{7}{4};$$

For(b) $$\int_1^2 \frac{3x+1}{6x} dx = \int_1^2 \tfrac{1}{2} + \frac{1}{6x} dx = \left[\tfrac{1}{2} x + \tfrac{1}{6} \ln |x| \right]_1^2 = \tfrac{1}{2} + \tfrac{1}{6} \ln 2.$$

End of Examination

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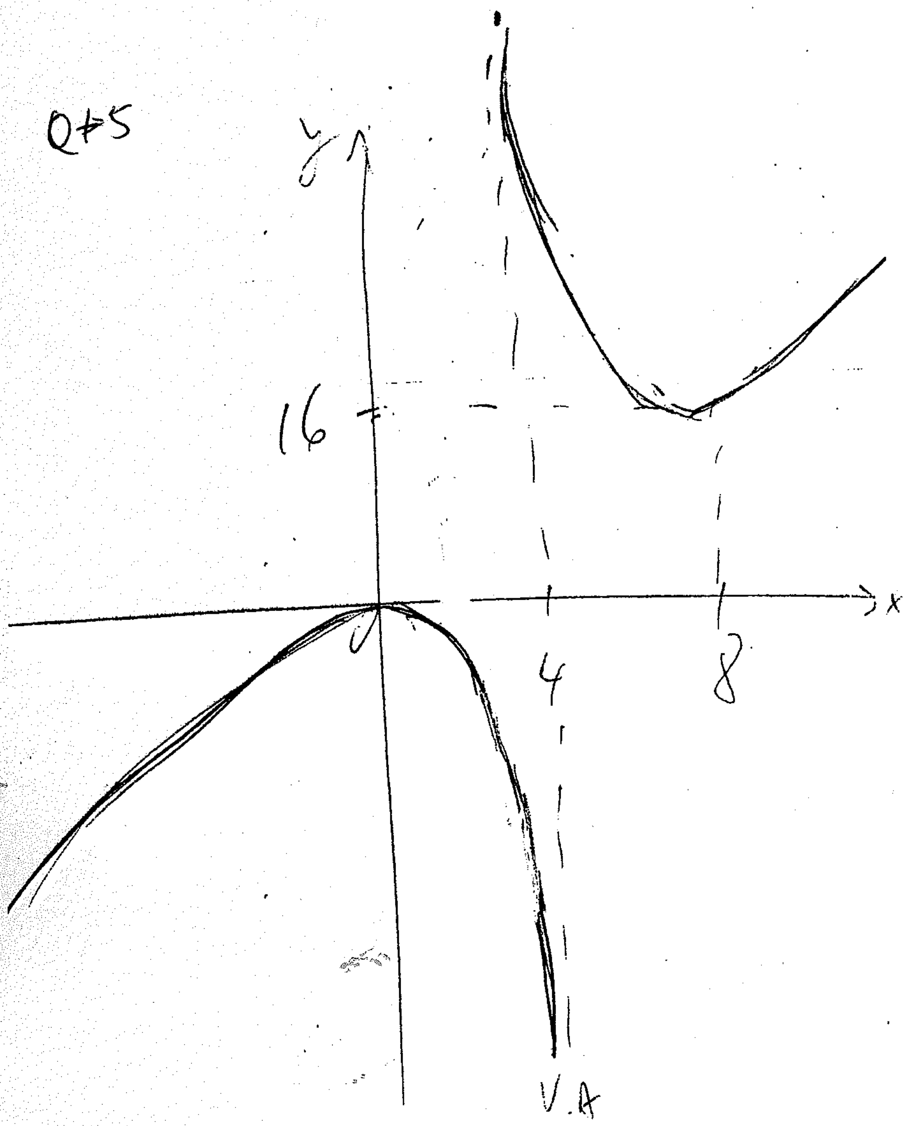

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