

THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
Math 249 L(01) - Fall, 2006

Time: 2 hours

NOTE: A calculator *is* allowed.

1. Find (a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{e^{x^2} - 1}$; (b) $\lim_{x \rightarrow +\infty} \frac{1 - \cos x}{e^{x^2} - 1}$; (c) $\lim_{x \rightarrow \infty} \frac{\ln x}{e^{-x^2} - 1}$.
2. Find the domain of definition and derivative of
(a) $f(x) = \frac{\ln(3x + 2)}{\sqrt{2 - x}}$; (b) $g(x) = x^{\cos x}$.
3. Find the tangent line approximation= linearization
(a) of $f(x) = \sqrt[3]{13 - 5x}$ around $x_0 = 1$;
then use it to estimate $\sqrt[3]{9}$.
4. A rectangular box with a square base and a square lid is to hold 12cm^3 .
Find the dimensions of the most economical box if the material for the base costs 4 cents per cm^2 , and the material for the sides and lid costs 2 cents per cm^2 .
5. For $f(x) = \frac{x^2}{x - 4}$ find (you may use $f'(x) = \frac{x^2 - 8x}{(x - 4)^2}$ $f''(x) = \frac{32}{(x - 4)^3}$)
 - (a) the domain, vertical and horizontal asymptotes;
 - (b) intervals where f is increasing, resp. decreasing;
 - (c) intervals where f is concave up resp. down;
 - (d) all local and absolute extrem, and the range;
 - (e) sketch the graph.
6. Sketch a graph of one function satisfying all the following conditions:
 - (a) f is defined on $(-\infty, 2]$;
 - (b) f is discontinuous at $x = -1, 0, 1$ and $\lim_{x \rightarrow 0} f(x) = 5$, $\lim_{x \rightarrow -1} f(x) DNE$ =does not exist;
otherwise continuous;
 - (c) $x = 1$ is a vertical and $y = 2$ is a horizontal asymptote;
 - (d) f is increasing on $(1, 2)$ and on $(-3, -2)$; it is decreasing on $(-1, 0)$, $(-2, -1)$ and on $(-\infty, -3)$;
and $f'(x) = 0$ for all $x \in (0, 1)$;

- (e) f is concave up on $(-1, 0)$, $(-4, -2)$ and concave down on $(-2, -1)$, $(-\infty, -4)$;
(f) absolute maximum value is 6, local minimum value is -2 .

7. Find (a) $\int x\sqrt{3x^2+2}dx$; (b) $\int \cos x \sin^2 x dx$.

8. Evaluate (a) $\int_1^2 \frac{6x}{3x+1} dx$; (b) $\int_1^2 \frac{3x+1}{6x} dx$.

End of Examination