MATH 249(L 01)
Midterm A

TIME: 55 minutes **Fall 2010**
SOLUTION

1. If $\cos\theta = -\frac{2}{5}$and $\pi < \theta < 2\pi$ find $\sin\theta$ and $\cot(2\theta)$.NO CALCULATORS!

   Since $\pi < \theta < 2\pi$ $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{4}{25}} = -\dfrac{\sqrt{21}}{5}$

   then $\cot(2\theta) = \dfrac{\cos(2\theta)}{\sin(2\theta)}$ using double angles formulae

   $$\cot(2\theta) = \frac{\cos^2\theta - \sin^2\theta}{2\cos\theta\sin\theta} = \frac{\frac{4}{25} - \frac{21}{25}}{2\cdot\left(\frac{-2}{5}\right)\left(\frac{-\sqrt{21}}{5}\right)} = \frac{-17}{4\sqrt{21}}.$$

2. For $y = \sin(\dfrac{x^2}{2})$ find the second derivative y''.

   using Chain Rule $y' = \left[\sin(\dfrac{x^2}{2})\right]' = \cos(\dfrac{x^2}{2})\cdot\left(\dfrac{1}{2}x^2\right)' = x\cos(\dfrac{x^2}{2})$

   then Product and Chain Rules $y'' = (x)'\cos(\dfrac{x^2}{2}) + x\left[\cos(\dfrac{x^2}{2})\right]' = 1\cdot\cos(\dfrac{x^2}{2}) + x\left[-\sin(\dfrac{x^2}{2})\cdot\left(\dfrac{1}{2}x^2\right)'\right]$

   $y'' = \cos(\dfrac{x^2}{2}) - x^2\sin(\dfrac{x^2}{2})$.

3. Find the limit $\lim_{x\to-\infty}\dfrac{\cos(2x^2+3)}{2x^2+3}$,if it exists.Explain what rule or theorem you used

   since the limit of the top DNE try to use Squeeze Th.

   since $-1 \le \cos(2x^2+3) \le 1$ $\dfrac{-1}{2x^2+3} \le \dfrac{\cos(2x^2+3)}{2x^2+3} \le \dfrac{1}{2x^2+3}$

   and $\lim_{x\to-\infty}\dfrac{\pm 1}{2x^2+3} = 0$ also $\lim_{x\to-\infty}\dfrac{\cos(2x^2+3)}{2x^2+3} = 0$.

4. Locate a root of $p(x) = x^5 + 3x^3 + 60$ i.e.

   check some values:

   $p(0) = 60$ $p(-1) = 54$ $p(-2) = 4$ positive $p(-3) =$ negative

   since $p(x)$ is continous we can apply IVT- Intermediate Value Theorem to the interval $[-3,-2]$

   and there is $c \in (-3,-2)$ such that $p(c) = 0$.

5. For

$$f(x) = \begin{cases} \sin(\frac{\pi}{2}x) & \text{for} \quad x > 3 \\ ax^2 + b & \text{for} \quad -1 \le x \le 3 \\ \cos\left(\frac{2\pi}{x}\right) & \text{for} \quad x < -1 \end{cases} .$$

we have to check the continuity at $x = 3$ and $x = -1$

$f(3) = 9a + b = \lim_{x \to 3^-} (ax^2 + 9) \qquad \lim_{x \to 3^+} \sin\left(\frac{\pi}{2}x\right) = \sin\frac{3}{2}\pi = -1 \to 9a + b = -1$

$f(-1) = a + b = \lim_{x \to -1^+} (ax^2 + 9) \qquad \lim_{x \to -1^-} \cos\left(\frac{2\pi}{x}\right) = \cos(-2\pi) = 1 \to a + b = 1$

now solve the system $9a + b = -1 \quad a + b = 1$ subtract the second from the first equation

$8a = -2 \qquad a = \frac{-1}{4} \qquad b = 1 - a = \frac{5}{4}.$

6. Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $[0, +\infty)$

(b) f is discontinuous at $x = 2, 3.4$ where $\lim_{x \to 2} f(x) = 0$, $\lim_{x \to 3} f(x)$ DNE(does not exist),otherwise continuous

(c) $x = 4$ is a vertical asymptote and $y = 2$ is a horizontal asymptote

(d) f is not differentiable at $x = 1, 2, 3, 4$ (no $f'(1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (4, 5)$, also $f'(5) = 0$.

(e) f is decreasing on $(1, 2)$ and $(5, \infty)$; f is increasing on $(0, 1)$,on $(2, 3)$and on $(3, 4)$

(f) the minimum value is -1. [10]

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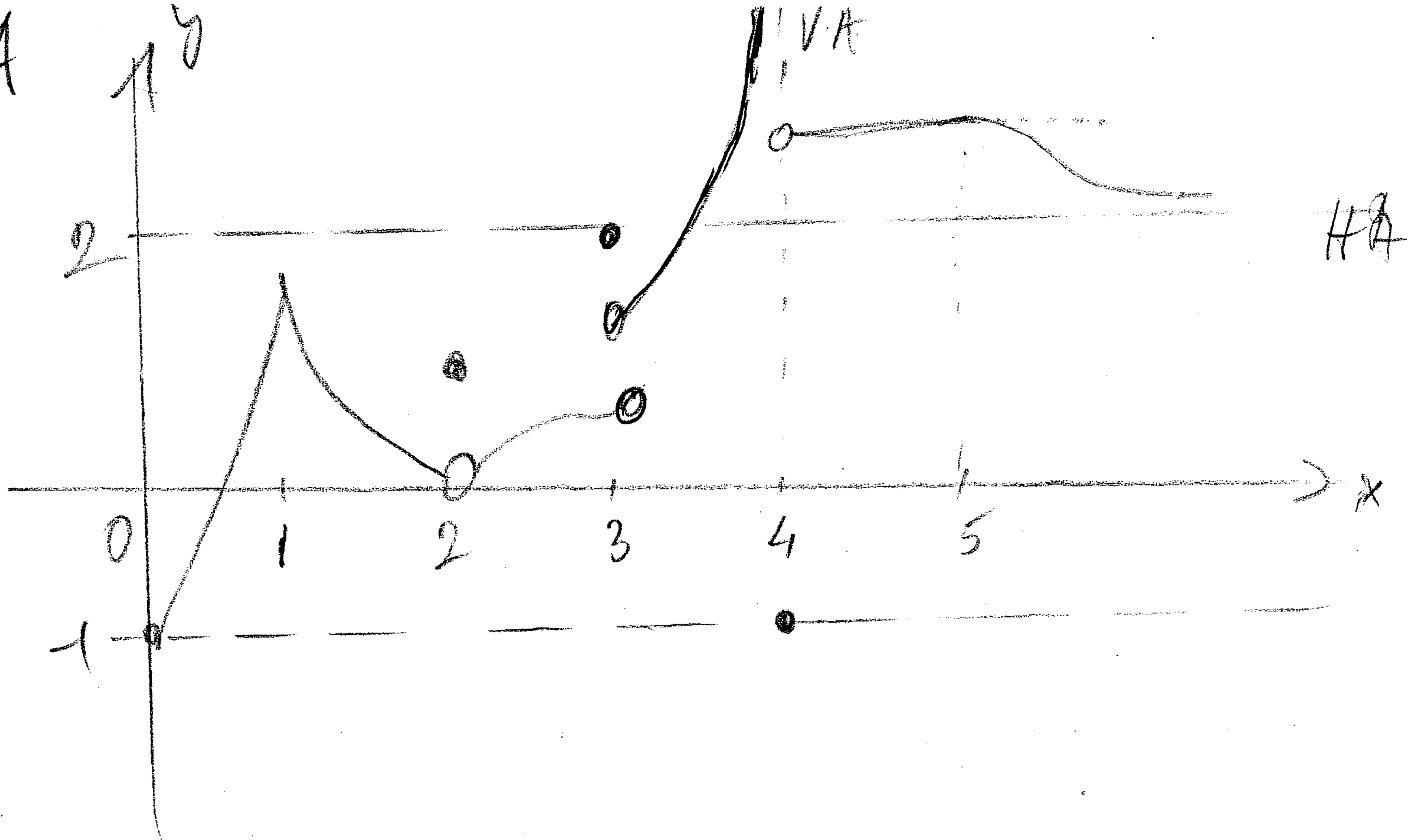

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