MATH 249(L 01)

Midterm B

Solution **Fall 2010**

1. If $\sin\theta = \dfrac{2}{3}$ nd $\dfrac{\pi}{2} < \theta < \pi$ find $\cos\theta$ and $\tan(2\theta)$.NO CALCULATORS!

   since $\dfrac{\pi}{2} < \theta < \pi$ $\quad \cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\frac{4}{9}} = -\dfrac{\sqrt{5}}{3}$

   then using double angle formulae

   $$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin\cos\theta)}{\cos^2\theta - \sin^2\theta} = \frac{2\cdot\frac{2}{3}\cdot\left(\frac{-\sqrt{5}}{3}\right)}{\frac{5}{9}-\frac{4}{9}} = -4\sqrt{5}.$$

2. For $\quad y = \cos\left(\dfrac{x^3}{3}\right)$ find the first derivative .by Chain Rule :

   $$y' = \left[cos\left(\frac{x^3}{3}\right)\right]' = -\sin\left(\frac{x^3}{3}\right)\cdot\left(\tfrac{1}{3}x^3\right)' = -\sin\left(\frac{x^3}{3}\right)\cdot\tfrac{1}{3}\cdot 3x^2 = -x^2\sin\left(\frac{x^3}{3}\right)$$

   then by Product and Chain Rules :

   $$y'' = (-x^2)'\sin\left(\frac{x^3}{3}\right) - x^2\left[\sin\left(\frac{x^3}{3}\right)\right]' = (-2x)\sin\left(\frac{x^3}{3}\right) - x^2\left[\cos\left(\frac{x^3}{3}\right)\cdot\left(\tfrac{1}{3}x^3\right)'\right] =$$

   $$= (-2x)\sin\left(\frac{x^3}{3}\right) - x^2\left[\cos\left(\frac{x^3}{3}\right)\cdot(x^2)\right] = -2x\sin\left(\frac{x^3}{3}\right) - x^4\cos\left(\frac{x^3}{3}\right).$$

3. Find the limit $\lim\limits_{x\to\infty}\dfrac{\sin(3x^2+2)}{3x^2+2}$,if it exists.Explain what rule or theorem you used

   since the limit of the top DNE try to use Squeeze Th.

   since $\quad -1 \le \sin(2x^2+3) \le 1 \qquad \dfrac{-1}{2x^2+3} \le \dfrac{\sin(2x^2+3)}{2x^2+3} \le \dfrac{1}{2x^2+3}$

   and $\lim\limits_{x\to-\infty}\dfrac{\pm 1}{2x^2+3} = 0$ also $\lim\limits_{x\to-\infty}\dfrac{\sin(2x^2+3)}{2x^2+3} = 0.$

4. Locate a root of $p(x) = 50 - 2x^3 - x^5$

   check some values:

   $p(0) = 50 \quad p(1) = 47 \quad p(2) = 2$ positive $\quad p(3) =$ negative

   since $p(x)$ is continous we can apply IVT- Intermediate Value Theorem to the interval $[2, 3]$

   and there is $c \in (2, 3)$ such that $p(c) = 0$.

5. For

   $$f(x) = \begin{cases} \cos(\frac{\pi}{2}x) & \text{for} \quad x < -3 \\ ax^2 + b & \text{for} \quad -3 \le x \le 6 \\ 2\sin\left(\dfrac{\pi}{x}\right) & \text{for} \quad x > 6 \end{cases}.$$

we have to check the continuity at $x = -3$ and $x = 6$

$f(-3) = 9a + b = \lim_{x \to 3^+} (ax^2 + 9)$ $\lim_{x \to 3^-} \cos(\frac{\pi}{2}x) = \cos(-\frac{3\pi}{2}) = 0 \to 9a + b = 0$

$f(6) = 36a + b = \lim_{x \to 6^-} (ax^2 + 9)$ $\lim_{x \to 6^+} 2\sin\left(\frac{\pi}{x}\right) = 2\sin\left(\frac{\pi}{6}\right) = 1 \to 36a + b = 1$

now solve the system $9a + b = 0$ $36a + b = 1$ subtract the first from the second equation

$27a = 1$ $a = \frac{1}{27}$ $b = -9a = \frac{-1}{3}$.

6. Sketch the graph of ONE function satisfying all the following conditions:

   (a) f is defined on $[-1, +\infty)$

   (b) f is discontinuous at $x = 1, 2.3$ where $\lim_{x \to 1} f(x) = 0$,
   $\lim_{x \to 2} f(x)$ DNE(does not exist),otherwise continuous

   (c) $x = 3$ is a vertical asymptote and $y = 1$ is a horizontal asymptote

   (d) f is not differentiable at $x = 0, 1, 2, 3$ (no $f'(0)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (3, 4)$ and $f'(5) = 0$.

   (e) f is decreasing on $(0, 1)$ and $(5, \infty)$;
   f is increasing on $(-1, 0)$,on $(1, 2)$and on $(2, 3)$

   (f) the minimum value is 0. [10]

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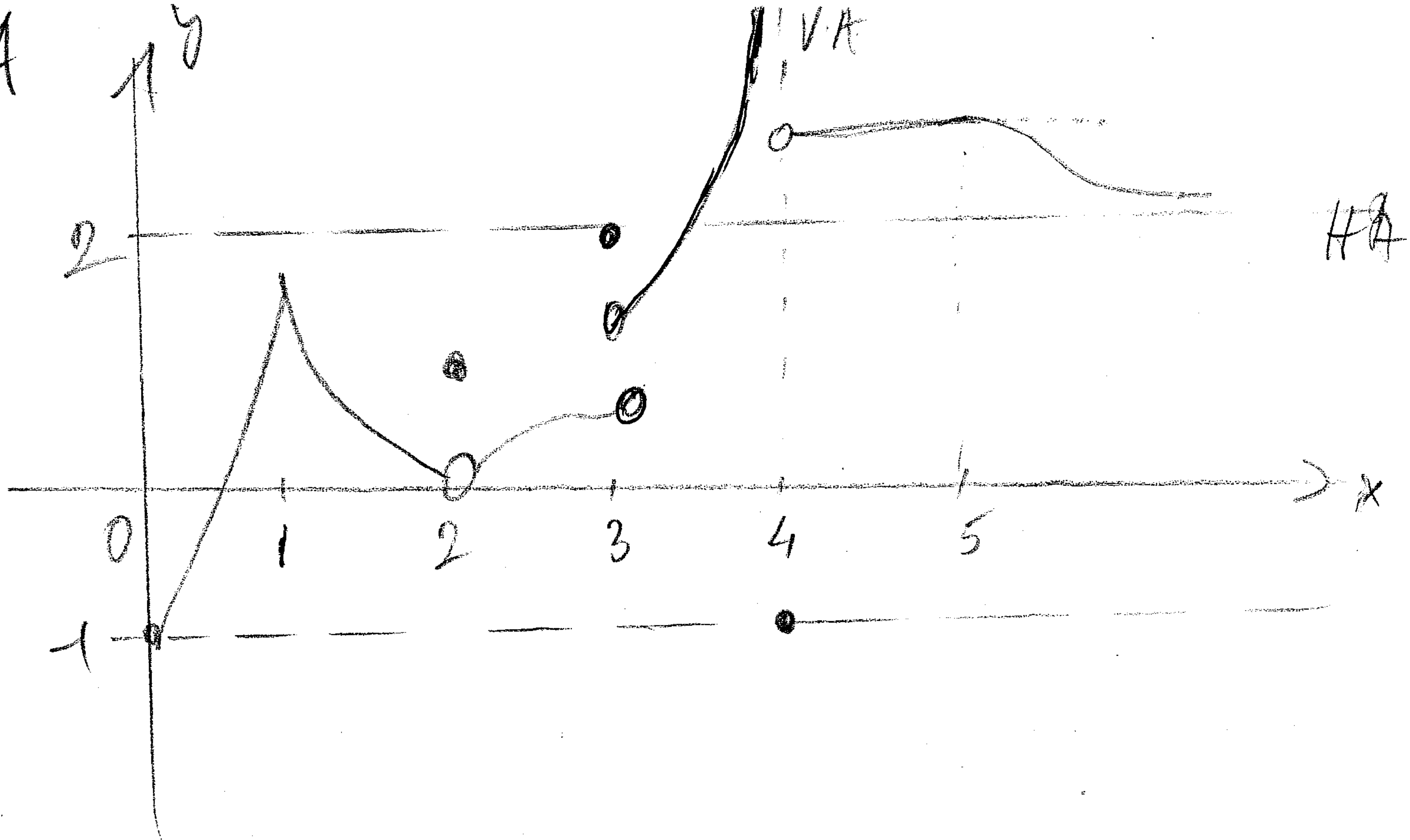

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