MATH 249
Midterm Handout

1. Evaluate

   $(a)\quad \lim_{x\to\infty}\left(x^2 - x^2\cos\frac{1}{x}\right) \qquad (b)\quad \lim_{x\to 0}\left(x^2 - x^2\cos\frac{1}{x}\right)$

2. Evaluate

   $(a)\quad \lim_{x\to 0}\frac{\sin x}{x-\pi} \qquad (b)\quad \lim_{x\to\pi}\frac{\sin x}{x-\pi} \qquad (c)\quad \lim_{x\to-\infty}\frac{\sin x}{x-\pi}.$

3. For $y = \dfrac{\cos \pi x}{1-x}$ find an equation of the tangent line at $x = -\dfrac{1}{2}$.

4. (a) For $y = \left(\sin\frac{1}{\sqrt{x^4+1}}\right)^3$ find y';

   (b) For $y = \cos(3-2x^2)$ find the second derivative y''.

5. Show that the function $f(x) = x - 2\sin(\pi x)$ has at least one positive zero i.e. $f(x) = 0$ at least for one $x > 0$.

6. Locate all 3 roots of $p(x) = 2x^3 - 6x^2 + 7$ i.e. find 3 intervals each containing one root.Sketch the graph of $y = p(x)$.

7. Find $\sec\theta$ if $\sin\theta = \frac{1}{5}$ and $\frac{\pi}{2} < \theta < \frac{3}{2}\pi$.NO calculator.

8. If $\cos\theta = \frac{2}{3}$and $\pi < \theta < 2\pi$ find $\sin\theta$ and then $\sin 2\theta$.No calculator.

9. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} \left(\frac{2}{2x+1} - 3\right)(4x^2-1) & \text{for} \quad x < -\frac{1}{2} \\ ax + b & \text{for} \quad -\frac{1}{2} \le x \le 2 \\ \cos(-\frac{\pi}{x}) & \text{for} \quad x > 2 \end{cases} .$$

10. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} \cos(\pi x) - 2\sin\frac{\pi x}{2} & \text{for} \quad x > 3 \\ ax^2 + b & \text{for} \quad 0 \le x \le 3 \\ 6\cdot\dfrac{\sqrt{9-x}-3}{x} & \text{for} \quad x < 0 \end{cases} .$$

11. **A**

    Sketch the graph of ONE function satisfying all the following conditions:

    (a) f is defined on $[-2, +\infty)$

    (b) f is discontinuous at $x = 0, 1, 2$ where $\lim_{x\to 1} f(x) = 3$, $\lim_{x\to 2} f(x)$ DNE(does not exist).otherwise continuous

    (c) $x = 0$ is a vertical asymptote and $y = -2$ is a horizontal asymptote

(d) f is not differentiable at $x = -1, 0, 1, 2$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (0, 1)$, also $f'(4) = 0$.

(e) f is increasing on $(-2, -1)$ and on $(2, 4)$; decreasing on $(-1, 0)$ and on $(4, +\infty)$;

(f) the maximum value is 3.

B

Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $(-\infty, 1]$

(b) f is discontinuous at $x = -3$ and $x = -2$ where $\lim\limits_{x \to -3^+} f(x) = f(-3) = 5$ otherwise continuous

(c) $x = -2$ is a vertical asymptote and $\lim\limits_{x \to -\infty} f(x) DNE$ (does not exists)

(d) f is not differentiable at $x = -1, -2, -3$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (-1, 0)$,also $f'(-4) = 0$;

(e) f is increasing on $(-3, -2)$ and on $(0, 1)$; decreasing on $(-2, -1)$;

(f) the minimum value is -2.

C

Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $(-\infty, 2)$

(b) f is discontinuous at $x = -3$ and $x = -2$ where $\lim\limits_{x \to -3} f(x) = 2$, and $x = -2$ is a vertical asymptote,otherwise continuous

(c) $y = 1$ is a horintal asymptote

(d) f is not differentiable at $x = -1, -2, -3$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (1, 2,)$also $f'(-4) = 0$;

(e) f is increasing on $(-1, 1)$ and on $(-3, -2)$; decreasing on $(-2, -1)$;

(f) the minimum value is $\frac{1}{2}$.

D

Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $(-1, \infty)$

(b) f is discontinuous at $x = 3$ and $x = 2$ where $\lim\limits_{x \to 2^+} f(x) = f(2) = 3$, $x = 3$ is a vertical asymptote ,otherwise continuous

(c) and $\lim\limits_{x \to +\infty} f(x) DNE$ (does not exists)

(d) f is not differentiable at $x = 0, 2, 3$ (no $f'(0)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (-1, 0)$,also $f'(4) = 0$.

(e) f is increasing on $(2,3)$ and on $(3,4)$; decreasing on $(0,2)$;

E

Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $(-\infty,+\infty)$

(b) f is discontinuous at $x=-1$ and $x=2$ where $\lim_{x\to-1} f(x)$ DNE
$x=2$ is a vertical asymptote ,otherwise continuous

(c) and $\lim_{x\to-\infty} f(x) DNE$ (does not exists), $y=-3$ is a horizontal asymptote;l

(d) f is not differentiable at $x=-1,1,2$ (no $f'(1)$) otherwise differentiable
and $f'(x)=0$ for all $x\in(2,3,)$also $f'(4)=0$;

(e) f is increasing on $(-1,1)$; decreasing on $(1,2)$ and on $(4,+\infty)$

(f) the maximum value is 4.