

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249-01 Quiz #1R Fall 2010

1. Solve for x:

$$|3x + 5| \leq |x - 5| \quad [3]$$

2. Solve for x $\frac{4}{x+3} \leq 1 - x.$ [3]

3. Find the radius of the circle $x^2 - 2x + y^2 + 6y = 0.$ [2]

4. Simplify $\frac{\frac{3x+2}{x} - 1}{\frac{x+1}{3}}$ and state from which x the given expression is defined. [2]

SOLUTION

For 1)

Since both sides are positive or 0 we can square:

$$(3x + 5)^2 \leq (x - 5)^2 \rightarrow 9x^2 + 30x + 25 \leq x^2 - 10x + 25$$

$$8x^2 + 40x \leq 0 \rightarrow 8x(x + 5) \leq 0 \quad \text{parabola open up with two roots}$$

$x = 0$ and $x = -5$ negative between roots, zero at the roots

OR

split points $x = -5, 0$, testing: $- \text{pos} - - -_{-5} - - - \text{neg} - - -_0 - - \text{pos} - -$

the solution is $x \in [-5, 0]$.

For 2)

$$\text{for } x \neq -3 \quad \frac{4}{x+3} - 1 + x \leq 0 \quad \rightarrow \frac{4 - x - 3 + x(x+3)}{x+3} \leq 0$$

$$\frac{x^2 + 2x + 1}{(x+3)} \leq 0 \quad \text{the polynomial on the top has a double root } \frac{(x+1)^2}{(x+3)} \leq 0$$

split points $x = -3, -1$

testing $- - \text{neg} - - -_{-3} - - - \text{pos} - - -_{-1} - - \text{pos} - -$

check the split points: $x = -1$ is also a solution $0 = 0$

so the solution set is $x \in (-\infty, -3) \cup \{-1\}$.

For 3)

$$\text{complete the squares} \quad x^2 - 2x + 1 + y^2 + 6y + 9 = 0 + 1 + 9$$

$$(x - 1)^2 + (y + 3)^2 = 10 \text{ so the centre is } C(1, -3) \text{ and } R = \sqrt{10}$$

For 4)

$$\text{for } x \neq -1, 0 \text{ (both)} \quad \frac{\frac{3x+2}{x} - 1}{\frac{x+1}{3}} = \frac{3x+2-x}{\frac{x+1}{3}} = \frac{2(x+1)}{x} \cdot \frac{3}{x+1} = \frac{6}{x}$$