The University of Calgary
Department of Mathematics and Statistics
MATH 249/01
Quiz # 1W

Fall 2010

Name:______________________________**I.D.#:**__________________

1. Solve for x: $|x-1| < 3\,|x|$. [3]

2. Solve for x: $\dfrac{10}{1-x} \leq x+4$. [3]

3. Find an equation of the circle with the center at the x-intercept of the line $2x-3y=6$ and radius 4. [2]

4. Simplify $\dfrac{x}{1-\frac{2}{x+1}}$ and state for which x the given expression is defined. [2]

Solution

For 1)

Since both sides are always positive or 0 we can square

$(x-1)^2 < (3x)^2 \;\to x^2-2x+1 < 9x^2 \to \qquad 0 < 8x^2+2x-1$

discriminant $D = 36 > 0$ so two roots $\qquad x = \frac{-1}{2} \cdot \frac{1}{4}$

$8(x-\frac{1}{4})\left(x+\frac{1}{2}\right) > 0 \qquad$ or $\qquad (4x-1)(2x+1) > 0$

parabola open up so negative between roots ,

OR $\qquad$ roots=split points $x = \frac{1}{4}, -\frac{1}{2}$

testing $\qquad - -^{pos} - -_{-\frac{1}{2}} - -^{neg} - - \tfrac{1}{4} - -^{pos} - -$

thus $\frac{1}{4} < x$ or $x < -\frac{1}{2} \qquad$ the solution is $\qquad x \in \left(-\infty, \dfrac{-1}{2}) \cup (\dfrac{1}{4}, +\infty\right)$

For 2)

For $x \neq 1$ everything on one side and common denominator:$\dfrac{10-(x+4)(1-x)}{(1-x)} \leq 0,$

simplify $\dfrac{10-(x+4-4x-x^2)}{(1-x)} \leq 0$ and finally $\qquad \dfrac{x^2+3x+6}{1-x} \leq 0$

the discriminant of the quadratic polynomial on the top is $D = 3^2 - 4 \cdot 6 = -15 < 0$

thus no real roots and the top is always positive.

The fraction will be negative (never 0) if the bottom is negative i.e.$1-x < 0, 1 < x$

OR using the split point method :the only one split point $x = 1$

testing: $-^{pos} - - - \circ_1 - -^{neg} - - -- \qquad$ so $x \in (1, +\infty)$

For 3)

We can find the x-intercept $(a, 0)$ by substituting $y = 0$ into the equation of the given line

so : $2x = 6 \qquad x = 3$, thus the center is at $C(3, 0)$ and an equation is

$(x-3)^2 + y^2 = 4^2$

For 4)

for $x \neq -1 \qquad \dfrac{x}{1 - \frac{2}{x+1}} = \dfrac{x}{\frac{x+1-2}{x+1}} = x \cdot \dfrac{x+1}{x-1} = \dfrac{x(x+1)}{x-1}$ for $x \neq \pm 1$