The University of Calgary
Department of Mathematics and Statistics
MATH 249-01 Quiz # 2R Fall,2010

1. For $f(x) = \sqrt{3-x}$ and $g(x) = \dfrac{x}{x-2}$ find $g \circ f$ and $g \circ g$ and their domains. [4]

2. For $f(x) = \dfrac{|x-1| - |x+1|}{x^2+x}$ find $\lim f(x)$ as

   (a) $x \to 0$ (b) $x \to -\infty$ (c) $x \to -1^+$ [3]

3. For $g(x) = \dfrac{x^2-x-2}{x^2-1}$ find $\lim g(x)$ as

   (a) $x \to -1$ (b) $x \to 1^-$ (c) $x \to +\infty$ [3]

Solution

For 1)

$g\circ f(x) = \dfrac{(..)}{(..)-2} = \dfrac{\sqrt{3-x}}{\sqrt{3-x}-2}$ for the domain solve $3-x \geq 0$ and $\sqrt{3-x}-2 \neq 0$

thus $3 \geq x$ and $\sqrt{3-x} \neq 2$ $3-x \neq 4$ $x \neq -1$ together $D_{g\circ f} = (-\infty, -1) \cup (-1, 3]$

also $g\circ f(x) = \dfrac{\sqrt{3-x}}{\sqrt{3-x}-2} \cdot \dfrac{\sqrt{3-x}+2}{\sqrt{3-x}+2} = \dfrac{\sqrt{3-x}\left(\sqrt{3-x}+2\right)}{(3-x)-2^2} = \dfrac{\sqrt{3-x}\left(\sqrt{3-x}+2\right)}{-(1+x)}$

$$g \circ g(x) = \frac{(..)}{(..)-2} = \frac{\frac{x}{x-2}}{\frac{x}{x-2}.-2} = \frac{\frac{x}{x-2}}{.\frac{x-2(x-2)}{x-2}} = \frac{\frac{x}{x-2}}{\frac{-x+4}{x-2}} = \frac{x}{x-2} \cdot \frac{x-2}{4-x} = \frac{x}{4-x}$$

for $x \neq 2, 4$

$D_{g\circ g} = \{x \neq 2, 4\} = (-\infty, 2) \cup (2, 4) \cup (4, +\infty)$

For 2)

for a) if x is close to 0 $|x-1| = |neg| = -(x-1) = 1-x$ and $|x+1| = |pos| = (x+1)$

so $\lim\limits_{x\to 0} \dfrac{|x-1|-|x+1|}{x^2+x} = \lim\limits_{x\to 0} \dfrac{1-x-(x+1)}{x^2+x} = \lim\limits_{x\to 0} \dfrac{-2x}{x(x+1)} = \lim\limits_{x\to 0} \dfrac{-2}{(x+1)} = -2$

for b) if x is "big neg." number $|x-1| = |neg| = -(x-1) = 1-x$ and $|x+1| = |neg| = -(x+1)$

so $\lim\limits_{x\to -\infty} \dfrac{|x-1|-|x+1|}{x^2+x} = \lim\limits_{x\to -\infty} \dfrac{1-x+(x+1)}{x^2+x} = \lim\limits_{x\to -\infty} \dfrac{2}{x(x+1)} = \text{"}\dfrac{2}{\infty}\text{"} = 0$

for c) x is close to -1 and $x > -1$ $x+1 > 0$

$|x-1| = |neg| = -(x-1) = 1-x$ and $|x+1| = |pos| = (x+1)$ as in a)

$\lim\limits_{x\to -1^+} \dfrac{|x-1|-|x+1|}{x^2+x} = \lim\limits_{x\to -1^+} \dfrac{1-x-(x+1)}{x(x+1)} = \lim\limits_{x\to -1^+} \dfrac{-2}{(x+1)} = \text{"}\dfrac{-2}{0^+}\text{"} = -\infty$

For 3).

simplify first, for $x \neq \pm 1$ $\quad g(x) = \dfrac{x^2 - x - 2}{x^2 - 1} = \dfrac{(x+1)(x-2)}{(x-1)\,(x+1)} = \dfrac{x-2}{x-1}$

thus **for a)** $\displaystyle \lim_{x \to -1} g(x) = \lim_{x \to -1} \frac{x-2}{x-1} = \frac{-3}{-2} = \frac{3}{2}$

for b) since $x < 1$ $\quad \displaystyle \lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} \frac{x-2}{x-1} = \text{"}\frac{-1}{0^-}\text{"} = +\infty$

for c) $\displaystyle \lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} \frac{x-2}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{1 - \frac{2}{x}}{1 - \frac{1}{x}} = 1$ since "$\frac{1}{\infty}$" $= 0$

also from the original form $\displaystyle \lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} \frac{x^2 - x - 2}{x^2 - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{1 - \frac{1}{x} - \frac{2}{x^2}}{1 - \frac{1}{x^2}} = 1$