The University of Calgary
Department of Mathematics and Statistics
MATH 249-01 Quiz # 2W Fall 2010

1. For $f(x) = \dfrac{x}{3-x}$ and $g(x) = \sqrt{2+x}$ find the compositions
   $f \circ g, \quad f \circ f$ and their domains. [4]

2. For $f(x) = \dfrac{x^2 - 4x + 3}{1 - x^2}$ find $\lim f(x)$
   (a) as $x \to 1$; (b)as $x \to +\infty$; (c) as $x \to -1^+$ [3]

3. For $g(x) = \dfrac{2 - \sqrt{5x-1}}{x-1}$ find $\lim g(x)$
   (a) as $x \to 1$; (b) as $x \to +\infty$ (c) as $x \to -\infty$. [3]

Solution

For 1)

$f \circ g(x) = \dfrac{(..)}{3-(..)} = \dfrac{\sqrt{2+x}}{3-\sqrt{2+x}}$ for the domain solve $2 + x \geq 0$ and $3 - \sqrt{2+x} \neq 0$

thus $x \geq -2$ and $3 \neq \sqrt{2+x}$ $\quad 9 \neq 2 + x \quad x \neq 7 \quad D_{f \circ g} = [-2, 7) \cup (7, +\infty)$

also $\quad f \circ g(x) = \dfrac{\sqrt{2+x}\left(3+\sqrt{2+x}\right)}{3^2 - (2+x)} = \dfrac{\sqrt{2+x}\left(3+\sqrt{2+x}\right)}{7-x}$

$$f \circ f\ (x) = \frac{(..)}{3-(..)} = \frac{\frac{x}{3-x}}{3 - \frac{x}{3-x}} = \frac{\frac{x}{3-x}}{\frac{3(3-x)-x}{3-x}} = \frac{x}{3-x} \cdot \frac{3-x}{9-4x} = \frac{x}{9-4x}$$

for $x \neq 3, \frac{9}{4}$ $\quad D_{f \circ f} = \left(-\infty, \frac{9}{4}\right) \cup \left(\frac{9}{4}, 3\right) \cup (3, +\infty)$

For 2)

simplify first ,for $x \neq \pm 1$ $\quad f(x) = \dfrac{x^2 - 4x + 3}{1 - x^2} = \dfrac{(x-1)(x-3)}{(1-x)(1+x)} = \dfrac{(x-1)(x-3)}{-(x-1)(1+x)} = -\dfrac{x-3}{1+x}$

thus **for a)** $\quad \lim_{x \to 1} f(x) = \lim_{x \to 1} -\dfrac{x-3}{1+x} = -\dfrac{-2}{2} = 1$

for b) $\quad \lim_{x \to \infty} f(x) = \lim_{x \to \infty} -\dfrac{x-3}{1+x} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} -\dfrac{1 - \frac{3}{x}}{\frac{1}{x} + 1} = -1$ (using $"\frac{1}{\infty} = 0)$

also from the original form $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{x^2 - 4x + 3}{1 - x^2} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to \infty} \dfrac{1 - \frac{4}{x} + \frac{3}{x^2}}{\frac{1}{x^2} - 1} = -1$

for c) $\quad$ since $x > -1$ $\quad \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} -\dfrac{x-3}{1+x} = "\dfrac{4}{0^+}" = +\infty$

For 3)

simplify first, for $x \neq 1$ and $5x-1 \geq 0$

$$g(x) = \frac{2-\sqrt{5x-1}}{x-1} \cdot \frac{2+\sqrt{5x-1}}{2+\sqrt{5x-1}} = \frac{2^2-(5x-1)}{(x-1)\left(2+\sqrt{5x-1}\right)} = \frac{5-5x}{(x-1)\left(2+\sqrt{5x-1}\right)} = \frac{-5}{\left(2+\sqrt{5x-1}\right)}$$

then **for a)**

$$\lim_{x\to 1} g(x) = \lim_{x\to 1} \frac{-5}{\left(2+\sqrt{5x-1}\right)} = \frac{-5}{\left(2+\sqrt{4}\right)} = \frac{-5}{4}$$

for b)

$$\lim_{x\to +\infty} g(x) = \lim_{x\to 1} \frac{-5}{\left(2+\sqrt{5x-1}\right)} = \text{"}\frac{-5}{\infty}\text{"} = 0$$

for c)

$\lim_{x\to -\infty} g(x) = DNE =$does not exist since we have $\frac{-5}{\left(2+\sqrt{neg}\right)}$

$i.e.$the function has no values,it is not defined for $x < \frac{1}{5}$ $(5x-1<0)$.