The University of Calgary
Department of Mathematics and Statistics
MATH 249-01 Quiz #3R FALL 2010

1. Using the **definition** of derivative find $f'(-2)$ if $f(x) = x + \dfrac{2}{x}$. [3]

2. Using Differentiation Rules find f' if $f(x) = \dfrac{3\sqrt[3]{x} - 5}{25 - 3x}$, also find the domain of the derivative. [3]

3. Find an equation of the tangent to $y = \left(\dfrac{x}{3} - \dfrac{3}{x}\right)\sqrt{3x}$ at $x = 3$. [4]

Solution

For 1)

first $f(-2) = -2 + \dfrac{2}{-2} = -3$ then

$$f'(-2) = \lim_{x\to -2} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x\to -2} \frac{x + \frac{2}{x} + 3}{x + 2} = \lim_{x\to -2} \frac{\frac{x^2 + 2 + 3x)}{x}}{\frac{x+2}{1}} = \lim_{x\to -2} \frac{(x+2)(x+1)}{(x+2)(x)} =$$

$$= \lim_{x\to -2} \frac{x+1}{x} = \frac{-1}{-2} = \frac{1}{2}$$

OR $$f'(-2) = \lim_{h\to -0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to -0} \frac{-2 + h + \frac{2}{-2+h} + 3}{h} =$$

$$= \lim_{h\to -0} \frac{1}{h}\left[\frac{(h-2)^2 + 2 + 3(h-2)}{-2+h}\right] = \lim_{h\to -0} \frac{1}{h}\left[\frac{h^2 - 4h + 6 + 3h - 6}{-2+h}\right] =$$

$$\lim_{h\to -0} \frac{1}{h}\left[\frac{h(h-1)}{-2+h}\right] = \lim_{h\to -0}\left[\frac{h-1}{-2+h}\right] = \frac{1}{2}$$

check byt Rules:

$$f'(x) = 1 - \tfrac{2}{x^2} \qquad f'(-2) = \tfrac{1}{2}$$

For 2)

For $x \neq \dfrac{25}{3}$ use Quotient and Power Rules

$$f'(x) = \frac{(3\sqrt[3]{x} - 5)'(25 - 3x) - (3\sqrt[3]{x} - 5)(25 - 3x)'}{(25 - 3x)^2} = \frac{\left(3 \cdot \frac{1}{3}x^{-\frac{2}{3}}\right)(25 - 3x) - (3\sqrt[3]{x} - 5)(-3)}{(25 - 3x)^2}$$

for $x \neq 0, \frac{25}{3}$ we can simplify $f'(x) = \dfrac{\frac{25}{x^{\frac{2}{3}}} + 6\sqrt[3]{x} - 15}{(25 - 3x)^2}$

For 3)

at $x = 3$ $\quad y = 0 \cdot \sqrt{9} = 0$ $\quad$ the point is $P(3, 0)$

so an equation of the tangent is $\quad y = m_t(x - 3)$

to find the slope you can simplify first :

$$y = \frac{x}{3} \cdot \sqrt{3}\sqrt{x} - \frac{3}{x} \cdot \sqrt{3}\sqrt{x} = \frac{\sqrt{3}}{3} x^{\frac{3}{2}} - 3\sqrt{3} x^{-\frac{1}{2}}$$

then $\quad y' = \frac{\sqrt{3}}{3}(x^{\frac{3}{2}})' - 3\sqrt{3}(x^{-\frac{1}{2}})' = \frac{\sqrt{3}}{3} \cdot \frac{3}{2} x^{\frac{1}{2}} - 3\sqrt{3}\left(-\frac{1}{2}\right) x^{-\frac{3}{2}} =$

$= \dfrac{\sqrt{3}}{2} x^{\frac{1}{2}} + \dfrac{3\sqrt{3}}{2x\sqrt{x}}$ at $x = 3 \qquad m = \frac{\sqrt{3}}{2}\sqrt{3} + \frac{3\sqrt{3}}{2} 3^{-\frac{3}{2}} = 2$

OR use Product and Chain Rules

$$y' = \left(\frac{1}{3}x - 3x^{-1}\right)\sqrt{3x} + \left(\frac{x}{3} - \frac{3}{x}\right)\left[(3x)^{\frac{1}{2}}\right]' = \left(\frac{1}{3} - 3\,(-1)\,x^{-2}\right)\sqrt{3x} + \left(\frac{x}{3} - \frac{3}{x}\right)\left[\frac{1}{2}\,(3x)^{-\frac{1}{2}} \cdot 3\right] =$$

$= \left(\dfrac{1}{3} + \dfrac{3}{x^2}\right)\sqrt{3x} + \left(\dfrac{x}{3} - \dfrac{3}{x}\right)\dfrac{3}{2\sqrt{3x}}$ at $x = 3 \qquad m_t = \left(\frac{1}{3} + \frac{1}{3}\right)\sqrt{9} + 0 = 2$

FINALLY $\qquad y = 2\,(x - 3) \qquad y = 2x - 6 \qquad 2x - y = 6$