The University of Calgary
Department of Mathematics and Statistics
MATH 249- 03 Quiz #3W FALL 2010

1. Using the **definition of derivative** find $f'(-1)$ if $f(x) = \sqrt{10 - x^2}$. [3]

2. using the rules of differentiation find $f'(x)$ if $f(x) = \dfrac{7x^2}{\sqrt[3]{x} + 3}$; .find the domain of the derivative. [3]

3. Find an equation of the tangent line to $y = 2\sqrt{x}(x^2 - 1) - \dfrac{x}{2}$ at $x = 4$. [4]

Solution
For 1)

$$f(-1) = \sqrt{9} = 3 \qquad f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \to -1} \frac{\sqrt{10 - x^2} - 3}{x + 1} \cdot \frac{\sqrt{10 - x^2} + 3}{\sqrt{10 - x^2} + 3} =$$

$$= \lim_{x \to -1} \frac{10 - x^2 - 3^2}{(x + 1)\left(\sqrt{10 - x^2} + 3\right)} = \lim_{x \to -1} \frac{1 - x^2}{(x + 1)\left(\sqrt{10 - x^2} + 3\right)} = \lim_{x \to -1} \frac{(1 - x)(1 + x)}{(x + 1)\left(\sqrt{10 - x^2} + 3\right)} =$$

$$= \lim_{x \to -1} \frac{1 - x}{\sqrt{10 - x^2} + 3} = \frac{2}{3+3} = \frac{1}{3}$$

OR

$$f'(-1) = \lim_{h \to 0} \frac{f(-1 + h) - f(-1)}{h} = \lim_{h \to 0} \frac{\sqrt{10 - (h - 1)^2} - 3}{h} =$$

$$= \lim_{h \to 0} \frac{\sqrt{10 - (h - 1)^2} - 3}{h} \cdot \frac{\sqrt{10 - (h - 1)^2} + 3}{\sqrt{10 - (h - 1)^2} + 3} = \lim_{h \to 0} \frac{1}{h} \left[\frac{10 - (h^2 - 2h + 1) - 9}{\sqrt{10 - (h - 1)^2} + 3} \right]$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{-h^2 + 2h}{\sqrt{10 - (h - 1)^2} + 3} \right] = \lim_{h \to 0} \left[\frac{-h + 2}{\sqrt{10 - (h - 1)^2} + 3} \right] = \frac{1}{3}$$

check by Rules $\quad f'(x) = \frac{1}{2}(10 - x^2)^{\frac{-1}{2}} \cdot (-2x) = \dfrac{-x}{\sqrt{10 - x^2}}$ at $x = -1$ we get 1/3.

For 2)

by Quotient and Power Rules $f'(x) = \left(\dfrac{7x^2}{\sqrt[3]{x} + 3} \right)' = \dfrac{(7x^2)'\left(\sqrt[3]{x} + 3\right) - 7x^2\left(\sqrt[3]{x} + 3\right)'}{\left(\sqrt[3]{x} + 3\right)^2} =$

$$= \frac{14x\left(\sqrt[3]{x} + 3\right) - 7x^2\left(\frac{1}{3}x^{-\frac{2}{3}}\right)}{\left(\sqrt[3]{x} + 3\right)^2} \text{ (we can simplify) } \frac{= 14x^{\frac{4}{3}} + 42x - \frac{7}{3}x^{\frac{4}{3}}}{\left(\sqrt[3]{x} + 3\right)^2} \text{ for } x \neq 0, -8$$

For 3)

at $x = 4 \qquad y = 58 \qquad$ so the point is $P(4, 58)$ an equation $\qquad y = m_t(x - 4) + 58$

for the slope : we can simplify first : $y = 2\sqrt{x}(x^2 - 1) - \dfrac{x}{2} = 2x^{\frac{5}{2}} - 2x^{\frac{1}{2}} - \frac{1}{2}x$

then $y' = 2 \cdot \frac{5}{2}x^{\frac{3}{2}} - 2 \cdot \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2} \cdot 1 = 5x\sqrt{x} - \frac{1}{\sqrt{x}} - \frac{1}{2}$ at $x = 4$

$m = 40 - 1 = 39 \qquad$ by Product and Chain Rules

$$y' = \left(2\sqrt{x}\right)'(x^2 - 1) + 2\sqrt{x}\,(x^2 - 1)' - \left(\frac{1}{2}x\right)' = 2 \cdot \frac{1}{2\sqrt{x}}(x^2 - 1) + 2\sqrt{x} \cdot 2x - \frac{1}{2} =$$

$= \frac{1}{\sqrt{x}}(x^2 - 1) + 4x\sqrt{x} - \frac{1}{2} \qquad$ at $x = 4 \qquad m = \frac{15}{2} + 32 - \frac{1}{2} = 39$

and $y = 39(x - 4) + 58$