The University of Calgary Department of Mathematics and Statistics
MATH 249 Lecture 01 Quiz #4R FALL 2010

1. Find an equation of the tangent line to
$$\frac{y^2}{2x} - 2\sqrt{xy} = 4 \quad \text{at the point } P(1,4). \qquad [4]$$

2. Find $\displaystyle\int \frac{3x^2 - 7x^3 + 5}{x^2} dx$ for $x \neq 0$. [3]

3. Solve $y^{'} = \cos(\pi - 2x)$, $y(\frac{\pi}{4}) = 1$. [3]

SOLUTION

For 1)

an equation is $y = m(x-1) + 4$, where $m = y'$ at the point P;
for y'use the implicit differentiation:

$$\frac{1}{2}\left[\frac{y^2}{x}\right]' - 2\left(\sqrt{xy}\right)' = (4)' \qquad \text{by Quotient and Chain Rules}$$

$$\frac{1}{2}\left[\frac{2yy^{'}x - y^2 \cdot 1}{x^2}\right] - 2 \cdot \tfrac{1}{2}(xy)^{-\frac{1}{2}}(xy)' = 0$$

$$\frac{1}{2}\left[\frac{2yy^{'}x - y^2}{x^2}\right] - (xy)^{-\frac{1}{2}}(y + xy') = 0$$

now for $x = 1, y = 4, \sqrt{xy} = 2, y' = m$ we get

$$\frac{1}{2}(8m - 16) - \frac{4+m}{2} = 0 \qquad 8m - m = 16 + 4$$

$7m = 20$, $\quad m = \dfrac{20}{7}$ and an equation of the tangent is

$$y = \frac{20}{7}(x-1) + 4 \qquad \text{OR} \qquad 7y - 20x = 8$$

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For 2)

$$\int \frac{3x^2 - 7x^3 + 5}{x^2} dx \qquad (\text{ get rid of the fraction})$$

$$= \int \frac{3x^2}{x^2}dx + \int \frac{-7x^3}{x^2}dx + \int \frac{5}{x^2}dx = 3\int dx - 7\int xdx + 5\int x^{-2}dx =$$

$$= 3x - 7 \cdot \tfrac{1}{2}x^2 + 5 \cdot \tfrac{x^{-1}}{-1} + c = 3x - \tfrac{7}{2}x^2 - \frac{5}{x} + c$$

For 3)

$y^{'} = \cos(\pi - 2x)$ using $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$

$$y = \int y^{'}dx = \int \cos(\pi - 2x)dx = \frac{1}{-2}\sin(\pi - 2x) + c$$

now the condition $x = \frac{\pi}{4}, y = 1$,solve for c ;

$$1 = \frac{-1}{2}\sin\left(\frac{\pi}{2}\right) + c = -\tfrac{1}{2} + c \qquad \tfrac{3}{2} = c$$

so for any x $\quad y = -\dfrac{1}{2}\sin(\pi - 2x) + \dfrac{3}{2}$