**The University of Calgary Department of Mathematics and Statistics**
**MATH 249/01 Quiz #4W SOLUTION FALL 2010**

1. Find an equation of the tangent line to

$$\frac{2x}{x+y} + 3xy^2 = -14 \quad \text{at the point } P(-1,2). \qquad [4]$$

2. Find $\int \left(2x - \frac{1}{2x}\right)^2 dx$ for $x \neq 0$. [3]

3. Solve $y' = \sin(\pi + 3x), \quad y(-\pi) = 1.$ [3]

**SOLUTION**

**For 1**)

an equation is $y = m(x+1) + 2$, where $m = y'$ at the point $P$.

for y'use implicit differentiation:

$$\left(\frac{2x}{x+y}\right)' + 3\left(xy^2\right)' = (-14)' \qquad \text{by Quotient and Product Rules}$$

$$2\left[\frac{1\cdot(x+y) - x\,(x+y)'}{(x+y)^2}\right] + 3\left(1\cdot y^2 + x\cdot 2yy'\right) = 0$$

$$2\left[\frac{x+y-x\,(1+y')}{(x+y)^2}\right] + 3y^2 + 6xyy' = 0$$

now for $x=-1, y=2, x+y=1, y'=m$ we get

$2\,(2+m) + 12 - 6\cdot 2m = 0 \qquad 16 = 10m$

$m = \frac{16}{10}, \quad m = \frac{8}{5}$ and an equation of the tangent is

$$y = \frac{8}{5}(x+1) + 2 \qquad \text{OR} \qquad 5y - 8x = 18$$

.

**For 2)**

$\int \left(2x - \frac{1}{2x}\right)^2 dx$ get rid of the power using $(A-B)^2 = A^2 - 2AB + B^2$

$$= \int \left(4x^2 - 2\cdot 2x\cdot \tfrac{1}{2x} + \frac{1}{4x^2}\right) dx = 4\int x^2 dx - 2\int dx + \tfrac{1}{4}\int x^{-2} dx =$$

$$= 4\cdot\frac{x^3}{3} - 2x + \tfrac{1}{4}\cdot\tfrac{x^{-1}}{-1} + c = \tfrac{4}{3}x^3 - 2x - \frac{1}{4x} + c$$

**For 3)**

$y' = \sin(\pi + 3x)$ using $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

$$y = \int y' dx = \int \sin(\pi+3x)dx = -\frac{1}{3}\cos(\pi+3x) + c$$

now the condition $x = -\pi, y = 1$,solve for c ;

$1 = \frac{-1}{3}\cos(-2\pi) + c = -\frac{1}{3} + c \qquad \frac{4}{3} = c$

so for any $x$. $\quad y = -\frac{1}{3}\cos(\pi + 3x) + \frac{4}{3}$